# Analytical investigation of phase detuning induced transparency in multi-ring cascaded filters

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implemented on such devices.

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*Abstract*—We propose a novel control strategy that introduces a transparent off-state in cascaded multistage filtering structures based on MicroRing Resonator elements. These are fundamental elements in integrated photonics and represent a widespread filtering solution, especially in higher-order structures, which allows the synthesis of high-performance filters. We showcase the effect that thermal control can have in such configurations to enable a transparent bar-state, suppressing the main resonant peaks. This effect is analyzed by assuming phase control over the rings compatible with the traditional thermal calibration setup

## I. INTRODUCTION

MicroRing Resonators (MRRs) represent one of the fundamental blocks of Photonics Integrated Circuits (PICs) design, seeing stand-alone implementation as simple filters or to introduce frequency-selective effects in more complex circuits. Their flexibility has made them popular solutions in many fields, ranging from optical signal processing and switching, up to neuromorphic and computing applications [1]. In high-performance filtering applications they are typically not present as a first-order standalone filtering element, but are coupled and cascaded in more complex structures, allowing spectral shaping and precises filter synthesis [2]: in particular, when deployed in even-order ladder networks, the allow the creation of flat-top steep roll-off filters, which are ideal for modern optical communications relying on Wavelength-Division Multiplexing (WDM), which require high performance filtering blocks.

Despite their multiple positive properties, MRR-based structure can also pose some challanges in implementing multichannel switching elements, due to their lack on hitless tunability: while through thermal control the resonant channel can be moved from the design frequency, there is not way to turn-off the resonance (Cross/Bar switching), leading to potential routing conflicts in channel dense WDM systems.

In this work, we investigate a control scheme to induce a transparent propagation state in multistage ladder filters[3], analyzing the losses and theoretical limitations of such a state. The proposed control does not require additional heating elements with respect to a traditional calibration setup, while providing a frequency-independent BAR transmission state.

Fig. 1. Circuit schematic for a two-stage second-order MRRs-based ladder filter.

#### II. MULTISTAGE LADDER MRR FILTERS

In our analysis we focus on a two-stage second-order ladder filter, depicted in Fig. 1: these structures are made of cascaded ring stages which are connected by two waveguide rails, introducing a phase shift between the different stages. In our analysis we limit the scope to symmetric structures, meaning that the waveguide-MRR coupling is equal inside the k stage ( $\kappa_{k,1} = \kappa_{k,3}$ ), so that each stage is defined by the waveguide-MRR and MRR-MRR coupling only. Assuming lossless coupling and propagation in the MRRs, the round-trip can be modelled considering only the phase effect. In the following equation the phase terms are expressed as  $\Phi_{k,(1,2)} = \frac{2\pi n_{\text{eff}}L_{k,(1,2)}}{\lambda}$  $\frac{L_{k,(1,2)}}{\lambda}$ , with round-trip length defined through the ring radius  $L = 2\pi R_{k,(1,2)}$ . The coupling factors are expressed as  $\kappa_{k,1}$ ,  $\kappa_{k,2}$ , which represents the waveguidering and ring-ring coupling respectively. The transmission coefficients t are therefore defined as  $t_{k,(1,2)} = \sqrt{1 - \kappa_{k,(1,2)}^2}$ The index  $k$  in all the previous and following equations is used to differentiate the stages of the structure, according to the numbering introduced in Fig. 1. The scattering matrix  $S^{(k)}$ for a second-order MRR filter with two arbitrary round-trip length can be analytically expressed as:

$$
S_{11}^{(k)} = \frac{t_{k,1} - t_{k,2}e^{j\phi_{k,1}} - t_{k,1}^2 t_{k,2}e^{j\phi_{k,2}} + t_{k,1}e^{j(\phi_{k,1} + \phi_{k,2})}}{1 - t_{k,1}t_{k,2}(e^{j\phi_{k,1}} + e^{j\phi_{k,2}}) + t_{k,1}^2 e^{j(\phi_{k,1} + \phi_{k,2})}}
$$

$$
S_{21}^{(k)} = \frac{-j\kappa_{k,1}^2 \kappa_{k,2}e^{j\frac{1}{2}((\phi_{k,1} + \phi_{k,2})}}{1 - t_{k,1}t_{k,2}(e^{j\phi_{k,1}} + e^{j\phi_{k,2}}) + t_{k,1}^2 e^{j(\phi_{k,1} + \phi_{k,2})}}
$$



Fig. 2. Drop response as a function of the antisymmetrical MRR phase shifts.

$$
S_{22}^{(k)} = \frac{t_{k,1} - t_{k,2}e^{j\phi_{k,2}} - t_{k,1}^2 t_{k,2}e^{j\phi_{k,1}} + t_{k,1}e^{j(\phi_{k,1} + \phi_{k,2})}}{1 - t_{k,1}t_{k,2}(e^{j\phi_{k,1}} + e^{j\phi_{k,2}}) + t_{k,1}^2e^{j(\phi_{k,1} + \phi_{k,2})}}
$$

Having defined the two stages of our filter as  $S^{(1)}$ ,  $S^{(2)}$ , the overall scattering matrix of the device can be modelled considering the  $\pi$  phase shift in the upper waveguide bus as  $S = S^{(2)} \Lambda S^{(1)}$ , with  $\Lambda = \text{diag}[-1, 1]$ :

$$
S_{11}=S_{21}^{(1)}S_{21}^{(2)}-S_{11}^{(1)}S_{11}^{(2)},\quad S_{21}=S_{21}^{(1)}S_{22}^{(2)}-S_{11}^{(1)}S_{21}^{(2)}
$$

With the analytical form derived, the simulation and parameter analysis for the device becomes trivial, although the rigorous derivation and analysis of the scattering matrix remains challenging from an analytical stand-point.

#### III. TRANSPARENT CONTROL TECHNIQUE

The proposed transparent state can be observed by introducing a difference in the round-trip phase between the two rings of stage[3], while also creating the mirror configuration in the second stage, compensating for the  $\pi$  waveguide bus shift. The maximum effect can be seen when the rings of the doublet are out-of-phase by a π factor, which can be obtained by heating either MRR #1 and #4, or #2 and #3: this leads to the maximum detuning between the resonant peaks, resulting in a configuration similar to a frequency-independent bar state. In this analysis the device has been simulated considering coupling parameters  $\kappa_{1,1} = 0.73$ ,  $\kappa_{1,2} = 0.1$ ,  $\kappa_{2,1} = 0.3$ ,  $\kappa_{2,2} = 0.2$ , chosen to produce a flat-top filter without in-band ripples, which can be seen in Fig. 2. The figure also showcases the drop response for the varying antisymmetrical phase shift between the rings, highlighting the transparent "off-resonant" state for a  $\pi$  shift. Due to the cumbersome scattering matrix formulation, rigorous analytical derivation of the full device transfer function does not provide more insight neither into the optimality of the solution, nor the boundary values of the response for the "transparent" state. The analytical derivation for the single stage can be carried out if constrained to the goal of minimizing the resonance, which simplifies the equations phase variables to  $\Phi_1 = 2n\pi$ ,  $\Phi_2 = 2n\pi + \Delta\Phi$ ,  $n \in \mathbb{N}$ , expressed as a function of the ring detuning  $\Delta\Phi$ . The single-



Fig. 3. Single-stage frequency response for the default and transparent state. stage drop response is therefore simplified to:

$$
S_{2,1} = \frac{-j\kappa_1^2 \kappa_2 e^{j\frac{\Delta \Phi}{2}}}{1 - t_1 t_2 - t_1 t_2 e^{j\Delta \Phi} + t_1^2 e^{j\Delta \Phi}}
$$

which is minimized for  $\Delta \Phi = \pi$  if  $t_1 \ll t_2$ , which is true for the design of the flat-top filters for the considered applications, due to the poles and zero placement requirements [4]. The minimization of the resonant peaks alone would not be a strong enough condition to ensure the transparent state, although the proposed control scheme assumes detuning between the rings of each doublet, as such the obtained magnitude represents the upper bound for the drop response (Fig. 3), which is shown to be  $S_{(2,1),\text{max}} = \kappa_2$ ,  $\Delta \Phi = \pi$ . The same analysis can be performed for the through port:

$$
S_{1,1} = \frac{t_1 - t_2 - t_1^2 t_2 e^{j\Delta\Phi} + t_1 e^{j\Delta\Phi}}{1 - t_1 t_2 - t_1 t_2 e^{j\Delta\Phi} + t_1^2 e^{j\Delta\Phi}}
$$

highlighting the lower bound of the response as  $S_{(1,1),min}$  =  $t_2$ ,  $\Delta \Phi = \pi$ . Taking into account the  $\pi$  phase shift introduced by the waveguide bus for the cascaded devices, the antisymmetric control required to create the transparent state in the multi-stage device become apparent: while the response of the single stage does not change between the cases  $\Delta \Phi = \pi$  and  $\Delta \Phi = -\pi$ , the alternated detuning (either MRR #1-#4 or #2-#3) is necessary to compensate for the additional phase shift.

## IV. CONCLUSION

We showcased a novel analysis into resonance-suppression in cascaded MRR structure to achieve a bar frequencyindependent transmission state, allowing switching ring structures into a transparent, or "off-resonance", state.

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