Modified Rate Equation Model Including the Photon-Photon Resonance

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***Abstract*—When the longitudinal confinement factor in an edge-emitting laser with two dominant longitudinal modes is treated as a dynamic variable, the modulation transfer function has an extra term. This term produces a supplementary photon-photon resonance in the modulation response at a frequency approximately equal to the frequency separation between the longitudinal modes.**

Keywords-modulation response; modulation transfer function; photon-photon resonance; rate equations

I. INTRODUCTION

The continuous rise in the optical communication transmission rates increases the demand for directly modulated lasers with high modulation bandwidth. The modulation response of edge-emitting lasers, restricted by the limits of the electron-photon resonance (EPR), can be substantially improved by employing the photon-photon resonance (PPR).

PPR has been observed in distributed Bragg reflector (DBR) lasers [1-2], in coupled-cavity-injection-grating lasers [3-4], and in passive-feedback lasers [5]. The travelling wave model developed for DBR lasers explains the occurrence of the PPR by the presence of a mode that is spectrally close to the main mode, leading to one of the optical modulation sidebands being resonantly amplified by the cavity [6]. Another investigation has shown that the PPR behaviour is dependent on the grating coupling coefficient, on the internal loss in the grating section, on the grating phase and on the grating end mirror reflectivity and phase [7]. Some of these effects have been explained as a compound cavity effect, being favoured by a substantial penetration of the optical field into the DBR section, but the travelling wave model and the explanation of the PPR as a compound cavity effect given are only partly explaining the experimental observations of PPR.

II. MODIFIED RATE EQUATIONS

Our model starts from the density rate equations [8]

$$\frac{dN}{dt} = \frac{\eta_i I}{qV} - (R_{sp} + R_{nr}) - v_g g N_p \tag{1}$$

$$\frac{dN_p}{dt} = \left(\Gamma v_g g - \frac{1}{\tau_p}\right) N_p + \Gamma R'_{sp} \tag{2}$$

where N is the electron density and N_p is the photon density. η_i is the internal quantum efficiency, I is the bias current, q is the electron charge, V is the active volume, R_{sp} is the rate of spontaneous emission, R_{nr} is the nonradiative recombination rate, v_g is the group velocity, g is the gain per unit length, Γ is the confinement factor, τ_p is the photon lifetime, and R'_{sp} is the rate of the spontaneous emission into the mode of interest.

The classical small-signal response is obtained by taking the differential of (1) and (2) and considering I, N, N_p and g as dynamic variables, while $\Gamma=\Gamma_{xy}\Gamma_z$ is assumed to be time-independent (or averaged), under the assumption that the optical frequency is much more higher than the variation frequency of the dynamic variables. When Γ is also treated as a dynamic variable, the differential of (2) gets an extra term $(N_p \cdot v_g \cdot g + R'_{sp}) \cdot d\Gamma$.

By assuming that the gain variation dg is affected both by carrier and photon density variations ($dg = a \cdot dN + a_p \cdot dN_p$) the differential rate equations, including the extra term, become

$$\frac{d}{dt}\begin{bmatrix} dN \\ dN_p \end{bmatrix} = \begin{bmatrix} -\gamma_{NN} & -\gamma_{NP} \\ \gamma_{PN} & -\gamma_{PP} \end{bmatrix} \begin{bmatrix} dN \\ dN_p \end{bmatrix} + \begin{bmatrix} \dfrac{\eta_i}{qV} dI \\ (N_p v_g g + R'_{sp}) d\Gamma \end{bmatrix} \tag{3}$$

where γ_{NN}, γ_{NP}, γ_{PN} and γ_{PP} are rate coefficients, as defined in [8]. By assuming the small-signal responses to a sinusoidal current modulation $dI=I_1 \cdot exp(j\omega t)$ as $dN=N_1 \cdot exp(j\omega t)$ and $dN_p=N_{p1} \cdot exp(j\omega t)$, as in [8], the small-signal photon density, including the influence of the extra term, is given by

$$N_{p1} = \frac{\eta_i I_1}{qV} \cdot \frac{\gamma_{PN}}{\Delta} + (N_p v_g g + R'_{sp}) \cdot \frac{(\gamma_{NN} + j\omega)}{\Delta} \cdot \frac{d\Gamma}{e^{j\omega t}} \tag{4}$$

where $\Delta = (\gamma_{NN} + j\omega)(\gamma_{PP} + j\omega) + \gamma_{NP}\gamma_{PN}$.

When the photon field is approximated as a sum of two dominant longitudinal modes with a phase difference that does not vary in time $d(\Delta\varphi)/dt = 0$, the confinement factor Γ can be written as

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$$\Gamma(t)=\frac{\Gamma_{xy}}{2L}\int_0^L\left(A_1\sin(\omega_1 t-k_1 z+\Delta\varphi)+A_2\sin(\omega_2 t-k_2 z)\right)^2 dz=\frac{\Gamma_{xy}}{2L}\Gamma_z(t) \quad (5)$$

where ω_i, k_i and A_i are the angular frequency, the wave number and the amplitude for the i^{th} mode.

Consequently, the modulation transfer function including the influence of the extra term resulted from the (space and) time variation of the confinement factor, can be written as

$$H(\omega)=\frac{\eta_i}{qV}\cdot\frac{\int_0^T \frac{\gamma_{PN}}{\Delta}dt}{\int_0^T dt}+\frac{1}{I_1\cdot\int_0^T dt}\cdot\int_0^T\frac{(\gamma_{NN}+j\omega)\cdot(N_p v_g g+R'_{sp})}{\Delta\cdot e^{j\omega t}}\cdot\frac{d\Gamma}{dt}dt \quad (6)$$

where T is the time interval for which the phase difference, $\Delta\varphi$, between the longitudinal modes is maintained. The first term in (6) resembles the traditional modulation transfer function, with γ_{PN} and Δ taken as time-dependent, while the second term is resulted from considering the (space and) time dependence of the confinement factor. This second term of the modulation transfer function introduces the supplementary PPR placed at a frequency equal with the frequency difference between the two dominant longitudinal modes. When the two dominant longitudinal modes are consecutive longitudinal modes and their separation is not substantially altered by detuned loading, the PPR frequency occurs at about the round-trip frequency, in agreement with the experimental results reported in [2-5].

III. Simulation Examples And Discussion

If the phase difference between the two dominant modes is not maintained constant for long enough the PPR peak does not appear and the PPR peak is more pronounced as T is longer (i.e. as the phase difference $\Delta\varphi$ is maintained constant for longer). Fig. 1 illustrates the modulation response calculated at I_{bias}=100 mA for a 1310 nm GaInAsN Fabry-Pérot (FP) laser with good direct modulation properties, when L is either 500 or 1000 μm, and A_1=A_2=1. The time step used in the simulations, which were based on (6), was $1/(20\omega_1)$ and the total number of simulation steps was $1\cdot10^7$. In FP lasers the separation between the EPR and PPR peaks is always high, since high-frequency EPR requires a short cavity while a close-enough PPR requires a long cavity. However, the most important reason why PPR is very unlikely in FP lasers is that these lasers do not provide any mechanism to maintain the phase difference between longitudinal modes for long enough.

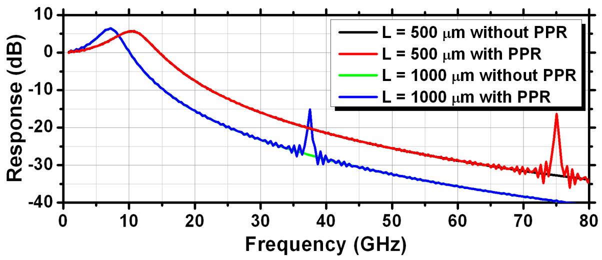

Figure 1. Modulation response for FP laser when L is 500 or 1000 μm.

A mechanism for phase-coupling between different longitudinal modes is associated with the presence of gratings so that distributed feedback (DFB) and DBR lasers with two longitudinal modes can exhibit PPR. Supplementary, in DFB and DBR lasers the mode spacing can be influenced by the coupling coefficient and detuned loading, which opens the possibility to obtain PPR at 30-40 GHz even with relatively short devices, which would also have a high-frequency EPR. It should be noted that our simple model does not take into account the coupling of the two longitudinal modes.

Fig. 2 presents the calculated modulation response when the frequency difference of the two dominant longitudinal modes is forced (for example by appropriate detuned loading) to 30 GHz for a 300 μm long DFB laser. The figure shows the modulation response calculated at I_{bias}=50 and 100 mA both with and without taking into account the second term of (6). It should be noted that the bias influence on mode spacing was not included in the calculations.

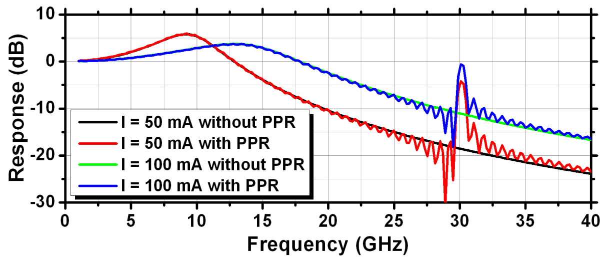

Figure 2. Modulation response for DFB laser when L is 300 μm.

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