Analysis of a Gap Plasmonic Waveguide Using the Frequency-Dependent 3-D LOD-FDTD Method

Jun Shibayama*, Ryoji Ando†, Junji Yamauchi‡ and Hisamatsu Nakano§
*Faculty of Science and Engineering
Hosei University, Tokyo 184-8584, Japan
Email: shiba@hosei.ac.jp
† Email: ryoji.ando.fb@stu.hosei.ac.jp
‡ Email: j.yma@hosei.ac.jp
§ Email: hymat@hosei.ac.jp

***Abstract*—A frequency-dependent implicit locally one-dimensional finite-difference time-domain (LOD-FDTD) method is developed for the analysis of three-dimensional (3-D) plasmonic structures. A brief formulation is given with the use of the simple trapezoidal recursive convolution technique. A gap plasmonic waveguide is analyzed to validate the 3-D LOD-FDTD. The computational time is significantly reduced to 60% of that of the traditional explicit FDTD.**

I. Introduction

Plasmonic waveguides have been extensively studied [1], [2], due to its ability to make a light wave propagate through subwavelength structures. For the analysis of plasmonic waveguides, the finite-difference time-domain (FDTD) method has widely been used, in which the frequency-dependent formulation is crucial to treat dispersive metals. Unfortunately, the spatial mesh size becomes quite small for achieving sufficient accuracy. This results in an extremely small time step with concomitant long computation time, when the traditional explicit FDTD is used. To efficiently analyze the plasmonic waveguides, we have developed an implicit locally one-dimensional (LOD) FDTD [3] using the simple and accurate trapezoidal recursive convolution technique for the frequency-dependent formulation [4]. Note that the frequency-dependent LOD-FDTD has been limited to the analysis of two-dimensional structures. In this article, we develop an LOD-FDTD method for the efficient analysis of three-dimensional (3-D) plasmonic structures.

II. Discussion

We consider the following Drude model [4]:

$$\varepsilon_r(\omega) = \varepsilon_\infty + \frac{\omega_\mathrm{p}^2}{j\omega\left(\nu_\mathrm{c} + j\omega\right)} \tag{1}$$

where ε_∞ is the dielectric constant of the material at infinite frequency, ω is the angular frequency, ω_p is the electron plasma frequency, and ν_c is the effective electron collision frequency. To take into account the dispersion property, we resort to the simple TRC technique [5], in which the linear polarization $\boldsymbol{P}$ is approximated using an average of the electric fields over two consecutive time steps. The TRC technique tailored for the 3-D formulation is expressed as

$$\boldsymbol{P}(n\Delta t) = \sum_{m=0}^{2n-1} \frac{\boldsymbol{E}^{n-m/2} + \boldsymbol{E}^{n-(m+1)/2}}{2}\chi^m \tag{2}$$

where

$$\chi^m = \int_{m\Delta t/2}^{(m+1)\Delta t/2} \frac{\omega_\mathrm{p}^2}{\nu_\mathrm{c}}\left(1 - e^{-\nu_c \tau}\right) d\tau.$$

Note that only a single convolution integral appears in (2), while providing the same accuracy as the piecewise linear RC technique with two convolution integrals.

As a result, we derive the following basic equations of the frequency-dependent 3-D LOD-FDTD for the first step (the normalized expression of field components is used):

$$E_x^{n+1/2} = \frac{\varepsilon_\infty - \chi^0/2}{\varepsilon_\infty + \chi^0/2}E_x^n + \frac{1}{\varepsilon_\infty + \chi^0/2}\phi_x^n + \frac{c\Delta t}{2\left(\varepsilon_\infty + \chi^0/2\right)}\left(\frac{\partial H_z^{n+1/2}}{\partial y} + \frac{\partial H_z^n}{\partial y}\right) \tag{3a}$$

$$E_y^{n+1/2} = \frac{\varepsilon_\infty - \chi^0/2}{\varepsilon_\infty + \chi^0/2}E_y^n + \frac{1}{\varepsilon_\infty + \chi^0/2}\phi_y^n + \frac{c\Delta t}{2\left(\varepsilon_\infty + \chi^0/2\right)}\left(\frac{\partial H_x^{n+1/2}}{\partial z} + \frac{\partial H_x^n}{\partial z}\right) \tag{3b}$$

$$E_z^{n+1/2} = \frac{\varepsilon_\infty - \chi^0/2}{\varepsilon_\infty + \chi^0/2}E_z^n + \frac{1}{\varepsilon_\infty + \chi^0/2}\phi_z^n + \frac{c\Delta t}{2\left(\varepsilon_\infty + \chi^0/2\right)}\left(\frac{\partial H_y^{n+1/2}}{\partial x} + \frac{\partial H_y^n}{\partial x}\right) \tag{3c}$$

$$H_x^{n+1/2} = H_x^n + \frac{c\Delta t}{2}\left(\frac{\partial E_y^{n+1/2}}{\partial z} + \frac{\partial E_y^n}{\partial z}\right) \tag{3d}$$

$$H_y^{n+1/2} = H_y^n + \frac{c\Delta t}{2}\left(\frac{\partial E_z^{n+1/2}}{\partial x} + \frac{\partial E_z^n}{\partial x}\right) \tag{3e}$$

$$H_z^{n+1/2} = H_z^n + \frac{c\Delta t}{2}\left(\frac{\partial E_x^{n+1/2}}{\partial y} + \frac{\partial E_x^n}{\partial y}\right) \tag{3f}$$

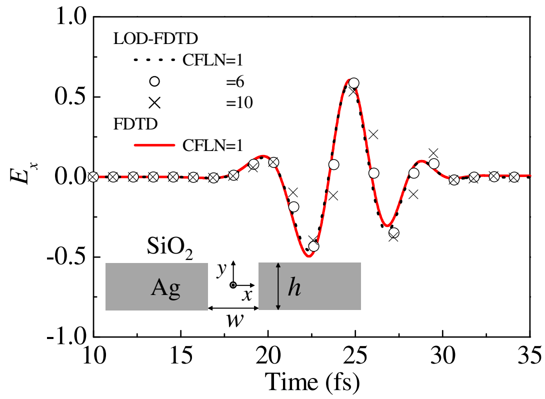

Fig. 1. Transient E_x component of the pulse.

where

$$\phi_\delta^n = \frac{E_\delta^n + E_\delta^{n-1/2}}{2}\Delta\chi^0 + e^{-\nu_c \Delta t/2}\phi_\delta^{n-1/2}$$

$$\Delta\chi^0 = -\frac{\omega_p^2}{\nu_c^2}(1-e^{-\nu_c\Delta t/2})^2$$

$$\chi^0 = \frac{\omega_p^2}{\nu_c}\left\{\frac{\Delta t}{2} - \frac{1}{\nu_c}(1-e^{-\nu_c\Delta t/2})\right\}$$

in which c is the speed of light in a vacuum, and δ is x, y or z. In the first step, we substitute (3d), (3e) and (3f) into (3b), (3c), and (3a), respectively, and then implicitly solve each resultant tridiagonal system of linear equations with the Thomas algorithm. After obtaining the electric field components $\boldsymbol{E}^{n+1/2}$, we explicitly calculate (3d)-(3f) for the magnetic field components $\boldsymbol{H}^{n+1/2}$. In the second step, although not shown, the equations can similarly be derived and solved as in the first step.

To assess the 3-D LOD-FDTD, we analyze a gap plasmonic waveguide illustrated in the inset of Fig. 1 ($h = w =$ 50 nm), in comparison with the TRC-based explicit FDTD. The dispersion of silver is taken into account by (1) with $\varepsilon_\infty = 3.7$, $\omega_D = 9.1$ eV, and $\nu_D = 0.018$ eV [4]. The refractive index of SiO_2 is 1.45. The sampling widths are $\Delta x = \Delta y = \Delta z = 5$ nm. The upper limit of the CFL condition is defined as Δt_{CFL} (= 0.00956 fs), and the ratio of $\Delta t/\Delta t_{CFL}$ as CFL number (CFLN).

The transient E_x component of the pulse with a center wavelength of $\lambda = 1.55$ μm is shown in Fig. 1. The pulse is observed at the center of the waveguide, where the observation plane is located 2 μm away from the incidence plane. For the LOD-FDTD, CFLN=1, 6, and 10 are used, while for the FDTD, CFLN=1 is used. The LOD result for CFLN=1 is found to be almost identical to the FDTD result, validating the 3-D LOD-FDTD. Even for the large CFLNs, the results are seen to be reasonably accurate. For CFLN=6, the computational time of the LOD-FDTD compares to that of the FDTD, while for CFLN=10, the time successfully reduces to 60%.

The snapshots of the E_x and H_y components for CFLN=10 are presented in Figs. 2(a) and (b), respectively, at the maximum field amplitude in Fig. 1. It is interesting to note that the H_x component widely extends, compared to the E_x component. They are in excellent agreement with the eigenmode fields used for the incident pulse, shown in Figs. 2(c) and (d). This indicates that the eigenmode fields accurately propagate along the waveguide even for a large CFLN.

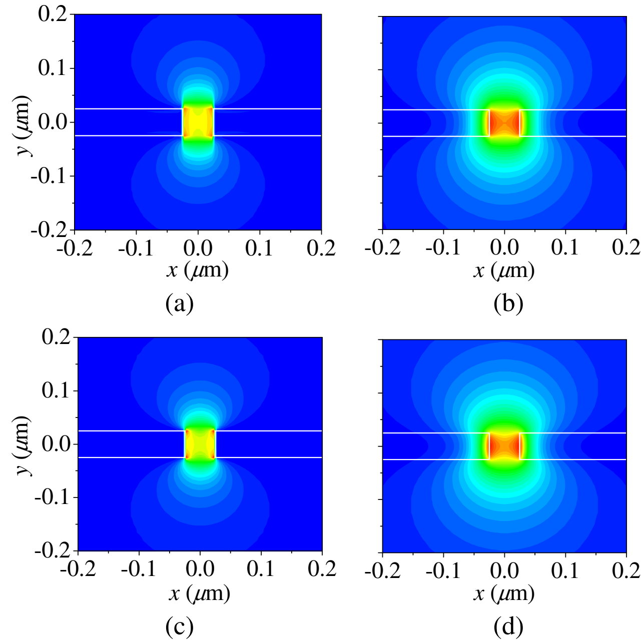

Fig. 2. Snapshots of the propagating pulse. (a) E_x and (b) H_y components. (c) and (d) are the corresponding eigenmode fields. Color represents intensity: red, highest; blue, lowest.

III. Conclusion

A frequency-dependent 3-D LOD-FDTD has been developed for the analysis of plasmonic devices. The effectiveness is investigated through the analysis of a gap plasmonic waveguide. It is shown that the acceptable numerical results are obtained even with a large time step beyond the stability criterion. The present formulation can also be extended to another dispersion models, such as Debye, Lorentz and their combination models.

References

[1] K. Tanaka and M. Tanaka, "Simulations of nanometric optical circuits based on surface plasmon polariton gap waveguide," *Appl. Phys. Lett.*, vol. 82, no. 8, pp. 1158-1160, Feb. 2003.

[2] G. Veronis and S. H. Fan, "Modes of subwavelength plasmonic slot waveguides," *J. Lightw. Technol.*, vol. 25, no. 9, pp. 2511-2521, Sep. 2007.

[3] J. Shibayama, M. Muraki, J. Yamauchi, and H. Nakano, "Efficient implicit FDTD algorithm based on locally one-dimensional scheme," *Electron. Lett.*, vol. 41, no. 19, pp. 1046-1047, Sep. 2005.

[4] J. Shibayama, A. Nomura, R. Ando, J. Yamauchi, and H. Nakano, "Frequency-dependent LOD-FDTD method and its application to the analyses of plasmonic waveguide devices," *IEEE J. Quantum Electron.*, vol. 46, no. 1, pp. 40-49, Jan. 2010.

[5] R. Siushansian and J. LoVetri, "A comparison of numerical techniques for modeling electromagnetic dispersive media," *IEEE Microw. Guided Wave Lett.*, vol. 5, no. 12, pp. 426-428, Dec. 1995.