Finite element simulation of optical modes in VCSELs

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***Abstract*—An important ingredient of VCSEL device simulation is the solution of the optical model. Challenges are the multiscale structure of realistic devices with thin layers in the Bragg mirror and active zone on the one hand and large total resonator volumes on the other hand. In order to compute the resonating modes and frequencies, Maxwell eigenvalue problems have to be solved on these geometries. Thereby, also layered infinite exterior domains have to be respected.**

In commercial simulation packages the optical problem is often restricted to geometries with cylindrical symmetry or even only simplified 1D material stacks are considered. However, with increasing complexity of devices and novel design ideas like inclusion of photonic crystal structures, full 3D modeling of VCSELs becomes very important.

In our contribution we present the finite element method for computation of the optical resonance modes and corresponding far fields in VCSELs. We investigate realistic 3D devices and quantify numerical effort and accuracy.

I. Introduction

The finite element method (FEM) is very well suited for simulation of nano-optical systems and devices [1], [2]. Its main features are the capability of exact geometric modeling due to usage of unstructured meshes and high accuracy at low computational cost. The finite element method offers great flexibility to approximate the solution: different mesh sizes h and polynomial ansatz functions of varying degree p can be combined to obtain high convergence rates. As a result, extremely demanding problems can be solved on standard workstations [3].

Solution of Maxwell's equations for realistic 3D VCSELs is a challenging task. Since the VCSEL resonator is realized by a distributed Bragg reflector (DBR), the geometry inherits a pronounced multiscale structure. Total device sizes are often up to several 100 cubic wavelengths with subwavelength DBR layer sizes, very thin active zones, and structured apertures. Furthermore, the structured infinite exterior of a VCSEL has to be modeled to obtain realistic predictions of radiation losses and lasing thresholds. As we will demonstrate, the FEM package JCMsuite offers a powerful tool for solution of this challenging simulation task.

II. Mathematical Background

The main physical effects in a VCSEL are associated to time scales, ranging over several orders of magnitude. Since the frequency of the optical modes is much higher than those of all other effects, a time-harmonic ansatz for the electric field is well-justified:

$$E(x,y,z,t) \quad = e^{-i\omega t}E(x,y,z), \tag{1}$$

where ω denotes the frequency. Using this ansatz in Maxwell's equations, the following second order "curl-curl equation" for the electric field can be derived:

$$\nabla \times \mu^{-1}\nabla \times E \quad = \omega^2 \varepsilon E. \tag{2}$$

In above equation no exterior current or charge density sources are present: the light field of a VCSEL is created by coupling to the electron system in the active layer. In Maxwell's equations this usually enters via the complex permittivity tensor ε. The resonance problem then consists of finding pairs (E,ω), such that Maxwell's equations (2) on the given geometry are fulfilled. Furthermore, the so called radiation condition has to be satisfied which requires that the resonance modes are purely outward radiating.

III. Finite Elements

For the finite element method the strong form of Maxwell's equations (2) has to be transformed into weak form by multiplication with a test function and integration over the computational domain. One arrives at the following formulation:

Find $(E,\omega) \in H\,(curl\;,\Omega) \times C$, such that:

$$a(E,v) = \omega^2 b(E,v)\,,\ \forall v \in H\,(curl\;,\Omega)\,, \tag{3}$$

with:

$$\begin{aligned} a(E,v) &= \int_\Omega \nabla \times v \cdot \mu^{-1} \cdot \nabla \times E \; dV, \\ b(E,v) &= \int_\Omega v \cdot \varepsilon \cdot E \; dV. \end{aligned}$$

In order to arrive at a finite dimensional problem which can be solved on a computer, the infinite dimensional function space $H\,(curl\;,\Omega)$ is restricted to a finite element space $V \subset H\,(curl\;,\Omega)$. The finite elements define so called shape functions whose support is restricted to individual patches of the triangulation of the geometry, see Fig. 1.

Finite element discretization of (3) leads to a sparse generalized eigenvalue problem which is solved numerically. Transparent boundary conditions are realized via perfectly matched layers (PML) [4].

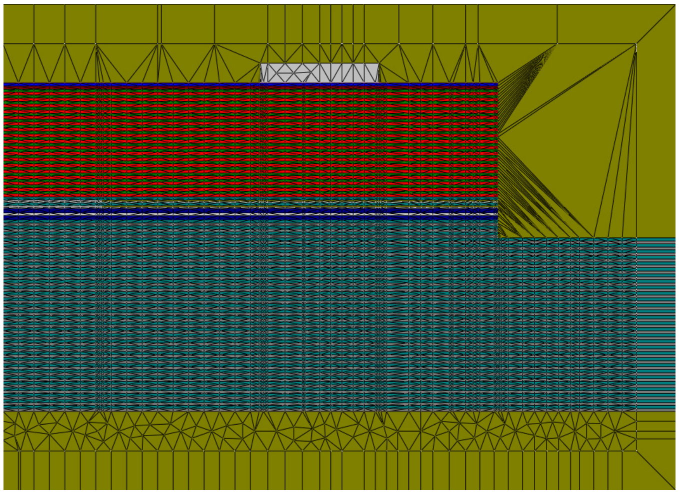

Fig. 1. Finite element triangulation of a VCSEL with cylindrical symmetry.

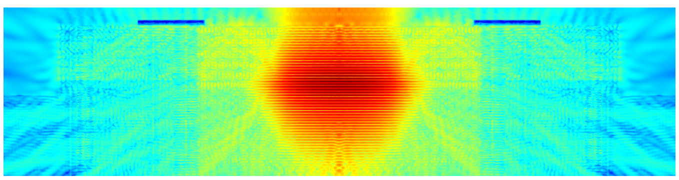

Fig. 2. Log of intensity of electric field of fundamental VCSEL mode obtained from FEM computation. The out-coupled field leaves the computational domain at the top boundary.

IV. VCSEL SIMULATION

Figure 1 shows the layout of the first VCSEL example we investigate. It inherits a rotational symmetry. The InGaAs active layer is embedded into a top and bottom GaAs/AlGaAs DBR mirror. Two AlOx apertures are placed above the active zone. Due to the cylindrical symmetry, the 3D resonance mode computation can be restricted to a 2D cross section. This leads to substantial savings in computational times and memory. Furthermore, the obtained solution can serve as a reference for a full 3D simulation and will allow us to quantify the accuracy of 3D simulations.

The fundamental resonating mode of the VCSEL computed with the cylinder symmetrical setup is shown in Fig. 2. The corresponding resonance frequency is given by:

$$\omega_{\text{3D,cyl.}} = 1.926 \cdot 10^{15} + 1.86 \cdot 10^{11} i, \qquad (4)$$

which corresponds to a wavelength of:

$$\lambda_{\text{3D,cyl.}} = 978.12\,\text{nm}.$$

which is close to the design wavelength of 980 nm.

Figure 3 shows the convergence of the real and imaginary part of the fundamental eigenvalue for different finite element degrees and mesh refinements. We observe that very high accuracies can be reached with relative errors down to 10^{-7} for the real part. Note that the relative error of the imaginary part which quantifies the gain/loss of the lasing mode is larger due its much smaller total value, c.f. (4). However, also here we reach relative accuracies well below 0.1%.

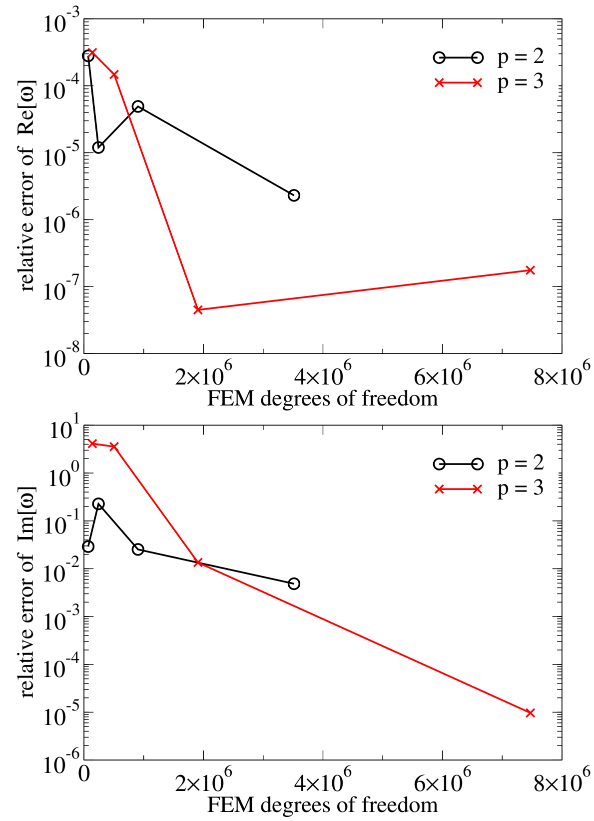

Fig. 3. Convergence of eigenmode computation: relative error of (a) real and (b) imaginary part of fundamental eigenvalue ω in dependence on number of FEM degrees of freedom for different polynomial orders p of the FEM approximation. Gain in active zone was fixed to $\Im m\,[\varepsilon_{\text{active}}] = 0.0076$. The relative error was computed against a FEM solution with $N = 13,876,992$ unknowns.

V. CONCLUSION

We have presented the finite element method for resonance mode computation in VCSELs. First numerical results for a rotationally symmetric device demonstrated fast convergence and high accuracy of the method. Real- and imaginary parts of the resonance frequency can be computed very accurately with relative errors down to 10^{-7}. As a next step we will present a comparison to a full 3D simulation and will also investigate more complex 3D device geometries.

REFERENCES

[1] M. Karl, B. Kettner, S. Burger, F. Schmidt, H. Kalt, and M. Hetterich, "Dependencies of micro-pillar cavity quality factors calculated with finite element methods," *Optics Express*, vol. 17, no. 1144, 2009.

[2] J. Hoffmann, C. Hafner, P. Leidenberger, J. Hesselbarth, and S. Burger, "Comparison of electromagnetic field solvers for the 3D analysis of plasmonic nano antennas," in *Numerical Methods in Optical Metrology*, vol. 7390. Proc. SPIE, 2009.

[3] S. Burger, L. Zschiedrich, and F. Schmidt, "Numerical investigation of optical resonances in circular grating resonators," in *Silicon Photonics V*, vol. 7606. Proc. SPIE, 2010.

[4] J. Bérenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.*, vol. 114, no. 2, pp. 185–200, 1994. [Online]. Available: http://dx.doi.org/10.1006/jcph.1994.1159