Asymptotic Analysis of Surface-Plasmon Whispering-Gallery Resonators for Large Q-factor

N. Cinosi[a], J. Sarma[b*], F. Causa[b]
(a) Computational Wave Group, Dept. of Mech. Eng., Imperial College, London, UK
(b) Dept. of Electronic & Electrical Engineering, Univ. of Bath, UK

***Abstract* - Large Q-factor resonators are essential components in most electronic and photonic systems; that requirement is even more pertinent to the next generation of highly compact nano-photonic integrated circuits. Surface-Plasmon waves combined with Whispering-Gallery modes have the potential to satisfy the dual requirements of small geometry and large Q. A quasi-analytic formulation for SPWG large Q resonator modes is presented in this paper and is demonstrated to be a convenient yet satisfactory procedure to compute device characteristics.**

I. Introduction

The phenomenon of surface electromagnetic waves, that of tight confinement of energy along the lateral axis at an interface while propagating along the longitudinal direction, has a long history and has found widespread application over the years, such as in radiowave propagation over the earth's surface, microwave propagation along dielectric coated metal wires, mm-wave and optical wave propagation in dielectric waveguides. More recently, however, the dominant interest has been in the study of Surface Plasmon waves (SP) which are a particular category of surface waves that are sustained at the interface between media with electrical permittivities of opposite sign - as is the case with a conventional dielectric and metal. SP waves have been used extensively to study the physics of surfaces, but the present trend is to achieve, for example, extremely sensitive bio-chemical detectors, nano-photonic circuits and devices for communications. The fact that very thin layers of metal deposited on a dielectric surface can conveniently and very compactly confine electromagnetic signals as SP waves from the tera-hertz up to the ultra-violet spectral range is the reason for the rapidly increasing interest in such structures for the next generation of nano-dimensional, potentially sub-wavelength, optical circuits and devices. One possible configuration to achieve a large Q-factor whilst maintaining a compact geometry is given by the disc resonator. The former characteristic can be realised using Whispering Gallery (WG) modes, while, for the latter feature, SP waves are considered to be appropriate, [1], [2].

That SPWG mode resonators are considered to be very important components in the next generation photonics devices and circuits is evident from the extensive study presented in [3], [4]. In both of those papers the resonator characteristics are evaluated in detail using numerical methods. The objective of this paper is to apply the asymptotic procedure, introduced for the analysis of WG modes [5], as a fast and efficient method to obtain the solutions for disc resonators with SP waves, that also provides more physical insight.

* This work was done when this author was with the University of Bath

II. Formulation and Analysis

The device geometry is shown in Fig. 1, with field variables approximated to be dependent only on the radial, r, and azimuthal, θ, axes. Quasi-scalar fields are assumed so that two dominant, independent polarisations have non-zero field components, { H_z, E_r, E_θ } and { E_z, H_r, H_θ } and a harmonic time dependence is implicit. The Helmholtz wave equation for the z-component of the field combined with the appropriate interface conditions at r = R for the field components is approximated to yield the modal dispersion relation for an individual mode in the form, [13]

$$n_A \frac{J_m\left(k_{mp} n_A R\right)}{J_m'\left(k_{mp} n_A R\right)} = n_B \frac{H_m^{(2)}\left(k_{mp} n_B R\right)}{H_m^{'(2)}\left(k_{mp} n_B R\right)}$$

where n_A and n_B are the refractive indices, J_m and H_m are Bessel and Hankel functions, m represents the order of azimuthal variation and p is that along the radial; k_{mp} is the wave number in vacuum corresponding to the complex resonant frequency ω_{mp}. The particular value of ω_{mp} for the corresponding resonant mode is what needs to be evaluated. Note that ω_{mp} must be taken to be complex since the above dispersion relation cannot otherwise be satisfied. Indeed, the complex ω establishes both the resonant frequency and the Q-factor for that mode.

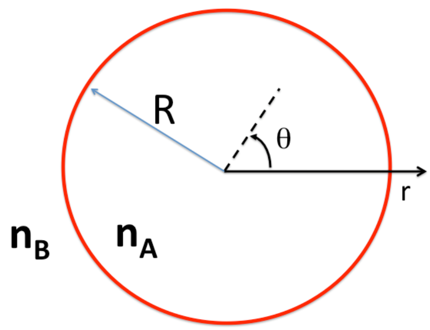

Fig.[1]: Geometry of the circular resonator analysed. R=5μm. For the case of dielectric/dielectric materials refractive indices are n_A=2.5, n_B=2.0. For metal/dielectric materials refractive indices are n_A=-i10, n_B=2.0.

The solution of the dispersion equation poses the main challenge. In general a totally numerical procedure must be followed beyond this point. However, for large Q conditions the resonator needs to operate with large m and in that case an approximation for Bessel and Hankel functions in terms of Airy functions can be used, which then results in a set of coupled polynomial equations that are solved numerically, [6], [7].

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III. Results

Using the method briefly outlined above, results are presented in Figs. 2 and 3 for SPWG resonators (i.e. r < R metal and r > R dielectric) and these are compared with those for 'traditional' WG (dielectric/dielectric) resonators.

Note from Fig.2 that WG resonators support modes of both polarisations while the SPWG modes are supported for only one polarisation ($H_z \neq 0$). The WG can support many radial-modes while the SPWG supports only one. Fig. 3 illustrates the mode shapes for the SPWG and WG structures.

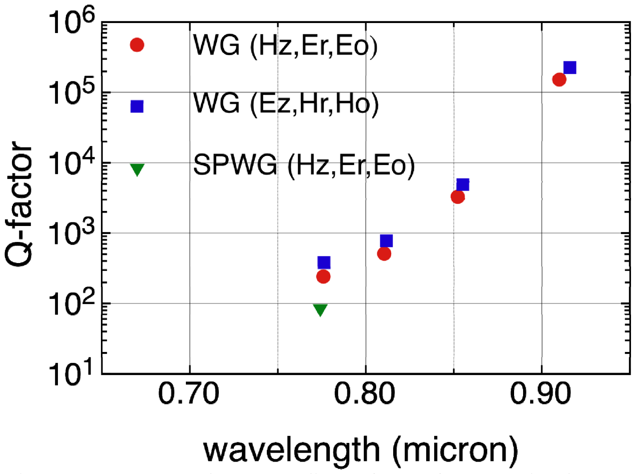

Fig.[2]: Resonances and corresponding Q-factor of a conventional WG resonator (m=80, p=1,2,3,4) and a SPWG resonator (m=80).

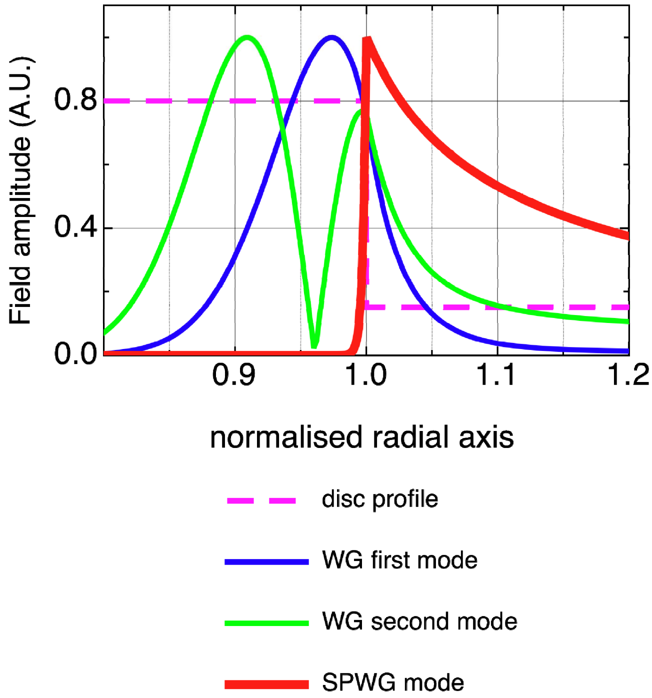

Fig.[3]: Field distribution of a conventional WG resonator (m=80, p=1,2) and a SPWG resonator (m=80).

IV. Summary

An asymptotic analysis is shown to provide acceptably satisfactory solutions for high Q-factor SPWG resonator modes

The presentation will provide details of the analysis and results for a large variety of device structures.

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