Analysis of FDTD Cloaking in the Visible Frequency Spectrum

Nadia Anam and Ebad Zahir
Department of Electrical and Electronic Engineering, American International University-Bangladesh, Banani, Dhaka-1213
Corresponding email: neela1303@yahoo.com

***Abstract*—This work proposes to study the behavior of different wavelengths of light in the visible spectrum when they are incident on a two-dimensional cylindrical structure that appears to be invisible or undetectable due to what is known as a 'cloak' formed around the actual object. This is achieved by modifying an existing Finite Difference Time-Domain model for such a structure in the Gigahertz range to function in the optical frequency region by changing different parameters such as the inner and outer radii of the cloak, the cell size and the number of time steps that affect the permeability, permittivity and hence refractive index of the structure. Results suggest that certain colors of light can be cloaked more effectively than the others.**

Keywords—cloaking; finite difference time-domain; metamaterial; optical frequency.

I. Introduction

The term cloaking has been famously extracted from books and movies where one could easily disappear from view by putting on a "magical cloak." In reality, this so-called-cloak design is possible by the use of artificially fabricated structures known as "metamaterials" [1]. The key reason for this requirement is that if an object needs to be undetectable, the light waves must bend around that object in such a way that when they arrive at the other end they remain undisturbed; such unusual angles of reflection cannot be achieved with materials that exist in nature. Unlike regular structures which are made from atoms and molecules, metamaterials are constructed from slightly larger elements which have a physical structure of their own [2].

II. Theoretical Background

The simulation of the proposed cloaking structure is based on the Finite Difference Time-Domain (FDTD) method which is able to solve Maxwell's equations in the time domain for complex structures and geometries [3]. The computational process becomes simpler without diminishing the accuracy of the outputs. The FDTD method considers the metamaterial models to be dispersive and hence the Drude Dispersive model may be used to characterize the negative permeability and permittivity of the material. In [4], a transverse magnetic wave generated from an infinite line source is considered to propagate in the z-direction (TM^z). The object to be cloaked is a cylindrical copper conductor surrounded by concentric rings of metamaterial structure. The propagating light waves are fully absorbed in the Perfectly Matched Layers (PML) leaving the computational domain without reflections. Periodic Boundary Condition (PBC) is applied to create the propagating wave as shown in Fig.1. The cloaking parameters are expressed as:

$$\mu_r(r) = \frac{r-r_a}{r},\ \mu_\varphi(r) = \frac{r}{r-r_a},\ \varepsilon_z(r) = \left(\frac{r_b}{r_b-r_a}\right)^2 \frac{r}{r-r_a} \tag{1}$$

where μ_r, μ_φ and ε_z represent dielectric and permeability values of the cylindrical co-ordinate system, r_a and r_b are the inner and outer radii and r is an arbitrary radius of the cloaking structure. The FDTD method calculates field distributions in a leap-frog manner where the electric or

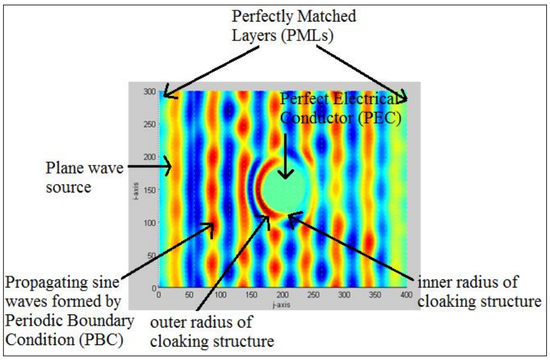

Fig.1. Simulation geometry for the FDTD method of cloaking

magnetic field at an instant is dependent on the past and future values of the field. The modified equation for electric field (*E*) suggested in [5]:

$$E_x^{n+1} = a_e(D_x^{n+1} - 2D_x^n + D_x^{n-1}) + b_e(D_x^{n+1} - D_x^{n-1}) + c_e(2E_x^n - E_x^{n-1}) + d_e(2E_x^n + E_x^{n-1}) + e_e E_x^{n-1} \tag{2}$$

where

$$a_e = \frac{4}{g},\ b_e = \frac{\gamma\,(2\,\Delta t)}{g},\ c_e = \frac{4\,\varepsilon_0\varepsilon_\infty}{g},\ d_e = \frac{\varepsilon_0\omega_p^2\,(\Delta t)^2}{g},\ e_e = \frac{\varepsilon_0\,\varepsilon_\infty\,\gamma_e(2\Delta t)}{g}$$

$$g = 4\varepsilon_0\,\varepsilon + \varepsilon_0\,\omega_p^2\,(\Delta t)^2 + \varepsilon_0\,\varepsilon\,\gamma\,(2\Delta t)$$

D is the electric flux density, $\omega_p$is the plasma frequency,γ isthe collision frequency and Δt is the size of each time step in the FDTD grid.

The magnetic flux *(H)* is obtained in a similar manner to as:

$$H_y^{n+1} = a_m(B_x^{n+1} - 2B_y^n + B_y^{n-1}) + b_m(B_y^{n+1} - B_y^{n-1}) + c_m(2H_y^n - H_y^{n-1}) + d_m(2H_y^n + H_y^{n-1}) + e_m H_y^{n-1} \tag{3}$$

For the TM^z wave, the auxiliary equations in (2) and (3) may be used to calculate the electric and magnetic fields E_v and H_v (where v ϵ x, y, z), from corresponding flux densities, D_v and B_v. The update equations for D_z, B_x and B_y are:

$$B_y^{n+\frac{1}{2}}[i+\tfrac{1}{2}, j] = B_x^{n-\frac{1}{2}}[i+\tfrac{1}{2}, j] + \frac{\Delta t}{\Delta}(E_z^n[i+1, j] - E_z^n[i, j]) \tag{4}$$

$$B_x^{n+\frac{1}{2}}[i, j+\tfrac{1}{2}] = B_x^{n-\frac{1}{2}}[i, j+\tfrac{1}{2}] + \frac{\Delta t}{\Delta}(-E_z^n[i, j+1] + E_z^n[i, j]) \tag{5}$$

$$D_z^{n+1}[i, j] = D_z^n[i, j] + \frac{\Delta t}{\Delta}\left(H_y^{n+\frac{1}{2}}[i+\tfrac{1}{2}, j] - H_y^{n+\frac{1}{2}}[i-\tfrac{1}{2}, j] - H_x^{n+\frac{1}{2}}[i, j+\tfrac{1}{2}] + H_x^{n+\frac{1}{2}}[i, j-\tfrac{1}{2}]\right) \tag{6}$$

III. Methodology

The outputs of the FDTD model are governed by a few prime parameters- the inner and outer radii of the cylindrical object r_a (=0.1m)and r_b(=0.2m), cell size in the x and y directions Δ (=3mm), the temporal steps Δt (=2.5 ps), the number of spatial steps in the i-j axes (=300), the perfectly matched layer (PML) widths on either ends of the FDTD grid space (=50 cells), the Courant Stability number S_c(= 0.25).The default frequency of the source is set at 2 GHz. To expand this model to optical frequency ranges, Δ had to be varied with respect to the wavelength as:

$$\Delta = \frac{\lambda}{50} \tag{7}$$

where λ is the wavelength of the light wave propagating from the source towards the cloaked object.

Consequently Δt has to be changed according to the relationship [6]:

$$S_c = u\frac{\Delta t}{\Delta} \tag{8}$$

where u is the speed of light in any medium, to ensure that the stability of the system remains unaltered.

Here the value of $u=c$ (=2.998x10^{8}ms^{-1}) since light is considered to be propagating through air. Lastly, r_a and r_b need to be scaled in the same ratio in which the value of Δ is varied. Since the frequency has been raised to hundreds of terahertz, the number of spatial steps in the i-j axes need to be lowered (=250) to compensate for the longer time needed by light to travel and get absorbed in the PML boundaries.

IV. Results

Table I contains relevant data of the modified optical FDTD system. The associated plots are illustrated in Fig. 2 to Fig. 4.

TABLE I. Variations in The Frequencies of Light in The Optical Range and Corresponding Outputs

Color of Light	f (THz)	Transmitted Field	Transmitted Field Beyond Slab	Transmitted Coefficient	Reflection Coefficient	Refractive Index (real)	Refractive Index (imaginary)
dark red	429	0.712	0.705	1.04	-0.04	1.9	0.5
red	451	0.698	0.698	1	0	1.8	0.5
orange	476	0.703	0.695	1.07	-0.07	1.95	0.56
yellow	500	0.713	0.698	1.02	-0.02	1.85	0.5
green	545	073	0.715	1.08	-0.08	1.97	0.58
aqua	612	0.705	0.703	1	0	1.8	0.45
blue	638	0.723	0.728	1.04	-0.04	1.9	0.51
indigo	706	0.675	0.708	1.06	-0.06	1.95	0.54
violet	750	0.72	0.74	1.03	-0.03	1.85	0.5

Fig. 2 shows the behavioral pattern of the plane wave before it is transmitted into the metamaterial cloak and after it has travelled beyond the cloaked object. At 4.5 THz (red) and 6.1 THz (aqua), the two transmitted fields overlap, indicating perfect cloaking. Maximum deviations (~5%) are obtained for indigo light. At optical range, the radius of the object is in the micrometer scale and thus it must be taken into account that as the size of the cloaked object is increased the percentage of inaccuracy may increase rationally. Fig.3 shows similar outcomes, with transmission and reflection coefficients of 1 and 0 respectively for red and aqua lights and unreliable results of 1.06 and -0.06 for indigo. Also, orange and green lights show divergences of around 7% and 8% respectively in Fig.4 but in Fig.3 these are somewhat lower. Fig.4 is a plot of the refractive index values (real and imaginary); these data are essential when one has to decide which metamaterial structures must be used for practical design purposes.

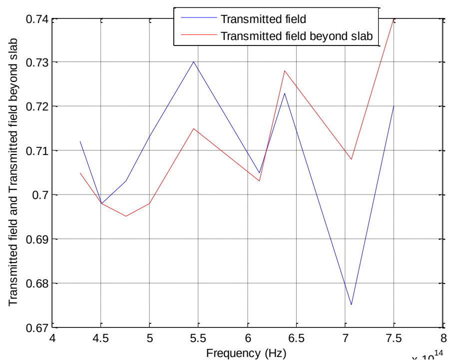

Fig.2. Plots for transmitted field before and beyond the cylindrical slab vs. frequency

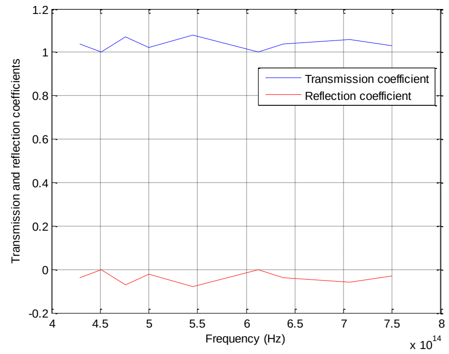

Fig.3. Plots for transmission and reflection coefficients vs. frequency

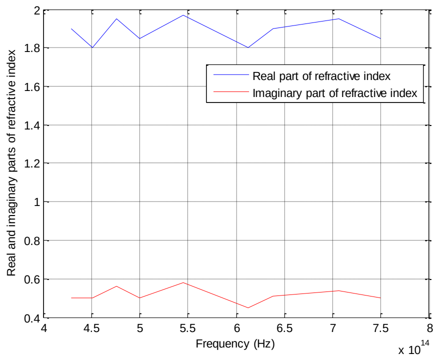

Fig.4. Plots for real and imaginary parts of refractive index vs. frequency

V. Conclusion

An existing FDTD model in the GHz range was expanded to optical frequencies by careful variation and scaling of some significant parameters within the simulation geometry to provide outputs for different colors of light in the visible range (from dark red with the lowest frequency to violet with the highest frequency). This transformation from the GHz to the THz range resulted in the dimension reduction of the cloaking structure from meters to micrometers. The intention was to deduce if different colors of light in the visible spectrum could be cloaked (i.e., travelled beyond an object and reflected back to the source location with minimum disturbance) consistently. Results suggested that red and aqua would be cloaked most accurately and blue, indigo and violet would be among the least preferred colors to be cloaked. Refractive index values suggest that in order to achieve nearer to perfect cloaking for the wavelengths yielding poor results, the polarization of the metamaterial cells or the metamaterial composition would require careful alteration.

References

[1] W. Cai andV. Shalaev, Optical Metamaterials Fundamentals and Applications, Springer, New York, pp. 3-4, 2010.

[2] J. Pendry, "Metamaterials and the Science of Invisibility," available at *https://www.youtube.com/watch?v=5ZOV_9Jirp0*, February 2014 [Accessed: 30 April, 2015].

[3] C. Argyropoulos, "FDTD Modelling of Electromagnetic Transformation Based Devices," PhD dissertation, Dept. of Electronic Engineering, Queen Mary, University of London, pp. 8,11, October 2010.

[4] A.Dawood, "Finite Difference Time–Domain Modelling of Metamaterials: GPU Implementation of Cylindrical Cloak," Advanced Electromagnetics, Vol. 2, No.2, pp. 10-17, 2013.

[5] Y. Zhao, C. Argyropoulos and Y. Hao, "Full-wave finite-difference time-domain simulation of electromagnetic cloaking structures," Optics Express, Vol. 16, Issue 9, pp. 6717-6730, April 2008.

[6] R. C. Booton, Jr, Computational Methods for Electromagnetics and Microwaves, USA: John Wiley and Sons, p.60-62, 1992.