Output Waveform Stability Analysis Integrated with Auto-Regressive Model in Two-Section DFB Self-Sustaining-Pulsation Lasers

Jer-Shien Chen*

**Intellectual Property Office, Ministry of Economic Affairs, Taipei, Taiwan. Email: jershienchen@jmail.com*

Abstruct We integrate Finite-Difference method and time series auto-regressive model to analyze two-section DFB self-sustaining-pulsation lasers periodic output stability. Systematic integrating such two methods in optical signals process is different to the existed researches.

I. Background and Introduction

Any optical network requires optical re-amplification, re-shaping, and re-timing for high-speed optical signals transmission. Re-timing technology needs an automatic and stable optical clock source to generate periodic waveforms which could achieve clock recovery for regeneration and to serve as the optical sampling pulses. Except electronic circuit oscillator, the self-sustaining-pulsation effect in a semiconductor laser can be used for generating optical clock signals, including multi-mode and single-mode types [1].

In the past, many research reports had observed various self-sustaining-pulsation phenomena from different types of semiconductor lasers, including using a two-section distributed feedback laser (TS-DFB) or additional electrical phase-tuning section. Self-sustaining-pulsation region is decided by tuning injection currents. Dispersive-Q-switching and mode beating are two major mechanisms governing the self-pulsation process [2]. However, the above existed prior results did not deal the following challenge, i.e., the self-sustaining-pulsation process about its stability. In other words, the output waveforms may not maintain stable sine wave. The output waveforms may emerge large amplitude in some periods, but emerge small amplitude in other periods. Besides, in short term process, the output waveforms may maintain stable sine wave, but, in long term process, the output waveforms may lose its stability, even the out environment does not change, i.e. using same laser chip and injecting certain constant currents.

In short, the major problem could be that it is necessary to develop one systematic model to find out those possible determinant factors of self-sustaining-pulsation output waveform stability. In this report, this point is unlike the existed reports which focus on how to generate periodic waveforms, not on how to maintain the stability of periodic waveforms.

II. Laser Structure Factors and Auto-Regressive Model

By the above issues in this report, there may be two levels of study the problem. First, analyzing the possible laser structure factors, in this report, first, the minor purpose is to study the relationship between periodic output waveforms stability and the design of TS-DFB takes different characteristics, for example, cavity length (L), linewidth enhancement factor (aH), and facet phase shift. This part could be simulated by finite-difference algorithm.

Second, the major purpose is to analyze the possible factor maintaining periodic output waveforms stability by inducing signal time series model, i.e. auto-regressive (AR) model. The signal auto-regressive model could be applied deeply not only with one previous stage, i.e. AR(1) model, but also with two previous stages, and three previous stages, etc., i.e. AR(2) model, AR(3) model. The AR(p) model has been applied in statistical signal processing, especially in signal estimation and detection. AR(p) model consider that for one dynamic feedback system, the next signal is not only affected by the system structure, but also affected by the previous signal. In other words, causality is existed between two signals, or even among signals. Because the self-sustaining-pulsation is a typical result in a dynamic feedback laser system, and based one signal time series model, we propose our research hypothesis (H1), and we choose some statistical characteristics listed in Table. I to construct AR(p) model. Table I summarizes our systematic process for output waveform stability analysis.

III. Finite-Difference Algorithm and AR(p) Regression Estimation

Several commercial tools have been proposed [3]; however, the information of internal field in a laser device may be confidential in such a commercial tool and it is not easy to using such tool to evaluate some stability condition; for example, in different iterations of one long term simulation, the middle result also may be covered and only final iteration result would been shown as simulation output. Which means the causality of signal time series can't be checked. Therefore, using finite-difference method, with Math lab code, is one way to simulate the optical field patterns and calculate the above two criteria of stability evaluation, which is based on the traveling-wave time/distance nonlinear partial differential equations of the laser, as shown in equation (1), wherein F{(T+1), (Z+1)} is forward waves in time and space, R{(T+1), (Z+1)} is backward waves, φ is facet phase shift, g is gain, δ is Bragg wavelength shift, k is grating coupling coefficient, and s is divided step in time and space.

$$\begin{bmatrix} F\{(T+1),(Z+1)\} \\ R\{(T+1),Z\} \end{bmatrix} = \left(\frac{2+gs-j\delta s-j\varphi}{2-gs+j\delta s+j\varphi}\right)\begin{bmatrix} \sqrt{1-\kappa^2 s^2} & j\kappa s \\ j\kappa s & \sqrt{1-\kappa^2 s^2} \end{bmatrix}\begin{bmatrix} F\{(T,Z)\} \\ R\{T,(Z+1)\} \end{bmatrix} \quad (1)$$

For a distributed feedback laser, we focus on the effects of cavity length, line width enhancement factor, and facet phase shift. Some selected structure parameters are given in Table. II and used in the simulation. Under practical application, laser cavity length is also taken into consideration, then, we choose 400um, 700um, and 1000um as short to long cavity lasers, and the two section length ratio is 1:1 for convenience.

In addition, so as to study the first level, different cases by tuning architecture parameters are listed in Table. III. We list 12 cases summary, i.e. Case A to Case L, and classify these cases into three group by level 1 factors listed in Table. II. Because paragraph limits, we only show Case F, Case G, Case K, and Case L in Fig. 1(a) to Fig. 1(d), respectively. Comparing Fig. 1(a) and Fig. 1(b), facet phase shift effect is demonstrated if only changing with/without this factor. Comparing Fig. 1(a) and Fig. 1(c), cavity length effect is demonstrated if only changing this factor. Comparing Fig. 1(c) and Fig. 1(d), linewidth enhancement effect is demonstrated if only changing this factor. Hence, in the proposed systematic procedure, we finished cross section output data analysis. However, in Level 1, cross section data only can be sure that cavity length, linewidth enhancement factor, and facet phase shift are important structures characteristics affecting self-sustaining-pulsation output waveform stability. It does not check the causality of remaining stability between signals. As shown in Fig. 2 for Case J, there are stable output waveforms in short transient state, but with unstable output in long term simulation. Without other laser structures changing, it is necessary to use Auto-regressive model to check the Level 2 problem, as shown in equation (2), wherein S(t) may be maximum, mean, standard deviation, and volatility; S(t) is next sampling signal respect to S(t-1); ε(t) is residual term, and β_i are coefficients. Limited to paragraph, we will only show the regression results with some statistical information: maximum, mean, standard deviation, and volatility. In addition, we include the above 12 cases to process Stata regression so that there are enough sample to be sure consistent results. Table. IV and V are the regression results. It is obvious that causality between sampling signals in both tables is with the statistical significance. Moreover, because the coefficient, β_1, β_2, β_3 and are less than 1, S(t) is also with convergence property, which means that causality between sampling signals would be remained, even in ling term simulation. In short, it is with relationship between self-sustaining-pulsation time series signals, and auto-regressive model might be one possible type.

$$\begin{cases} \text{AR(1) model: } S(t)=\beta_0+\beta_1 S(t-1)+\varepsilon(t) \\ \text{AR(2) model: } S(t)=\beta_0+\beta_1 S(t-1)+\beta_2 S(t-2)+\varepsilon(t) \\ \text{AR(3) model: } S(t)=\beta_0+\beta_1 S(t-1)+\beta_2 S(t-2)+\beta_3 S(t-3)+\varepsilon(t) \end{cases} \quad \ldots\ldots(2)$$

Table. I The Proposed Systematic Procedure

	Data Type	Multiple Criteria
Level 1	Cross Section Data	Different laser structures could be viewed as cross-section data with different characteristics, for example, cavity length (L), linewidth enhancement factor (aH), and facet phase shift.
		Finite-Difference Algorithm with Matlab code could be applied in simulating output waveform.
Level 2	Longitudinal Data	Self-sustaining-pulsation output signals could be viewed as longitudinal-section time series data.
		Auto-regressive model (AR(p)) of statistical characteristics, for example, maximum, mean, standard deviation, and standard deviation to mean ratio (volatility), with Stata could be applied in causality between time series signals.
		H1: the self-sustaining-pulsation is with time series auto-regressive property.

TABLE. II Selected Physical Parameters for TS-DFB Laser

	Symbol	Description	Quantity
Level 1 Factors	L	Cavity length	400um, 700um, 1000um
	aH	Linewidth enhancement factor	3, 4, 5
		1/4 Facet phase shift	With / Without
Other parameters	g_n	Differential gain	$3{*}10^{-16}$ cm^2
	N_{tr}	Carrier density at transparency	$1.5{*}10^{18}$ cm^{-3}
	λ_B	Free space wavelength	1550 nm
	ε	Nonlinear gain coefficient	$1{*}10^{-17}$ cm^3
	n_{effg}	Group refractive index	3.7
	Γ	Confinement factor	0.35
	k	Grating coupling coefficient	3000 1/m
	a	Loss	3000 1/m
	A	Linear recombination	$1.3{*}10^{-29}$ cm^3/s
	B	Bimolecular recombination	$1{*}10^{-10}$ cm^3/s
	C	Auger recombination	$1.3{*}10^{-28}$ cm^6/s

Table.III Different cases of stability analysis by tuning parameters

	L (μm)	aH	w/o 1/4 phase-\ shift	Brief description of optical spectrum, output waveform, and optical patterns
Case A	400	5	without	stable output with short transient state.
Case B	400	6	without	stable output with long term state.
Case C	700	3	without	stable output with long term state.
Case D	700	3	with	stable output with long term state.
Case E	700	4	without	stable output with short transient state.
Case F	700	5	without	unstable output with long term simulation; also shown in Fig. 1(a).
Case G	700	5	with	stable output with long term state; also shown in Fig. 1(b).
Case H	1000	2	without	stable output with long term state.
Case I	1000	3	without	stable output with long term state.
Case J	1000	3	with	stable output with short transient state; but unstable output with long term simulation; also shown in Fig. 2.
Case K	1000	4	without	approximated stable output with long term state; also shown in Fig. 1(c).
Case L	1000	5	without	unstable output with long term simulation; also shown in Fig. 1(d).

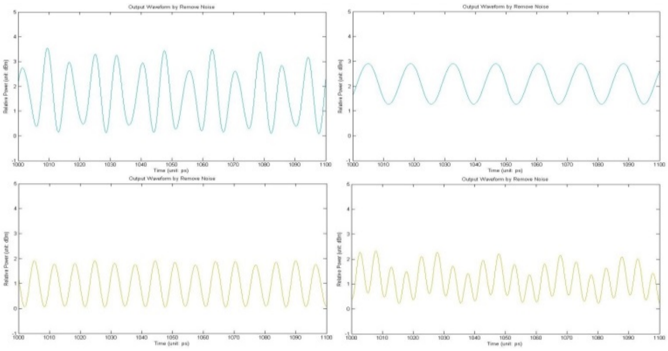

Fig.1(a)~1(d) (left-up for Case F, right-up for Case G, left-bottom for Case K, and right-bottom for Case L): calculated details of output waveform, besides vertical axis is relative output power, dBm, horizontal axis is calculation time, pico-second.

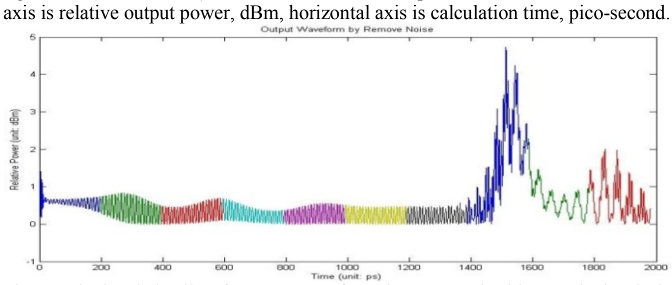

Fig.2: calculated details of output waveform in Case J, besides vertical axis is relative output power, dBm, horizontal axis is calculation time, pico-second.

TABLE. IV AR(P) MODEL BY "MEAN" AND "MAXIMUM" SAMPLES

	Using "Mean" as sample			Using "Maximum" as sample		
	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
S (t-1)	0.9913 ***	0.5917 ***	0.4545 ***	0.9576 ***	0.74456 ***	0.5182 ***
	(0.0125)	(0.0910)	(0.1128)	(0.0230)	(0.0810)	(0.1066)
S (t-2)		0.3998 ***	0.2364 **		0.2302 ***	0.2809 **
		(0.0909)	(0.1190)		(0.07984)	(0.1144)
S (t-3)			0.2967 **			0.1742 *
			(0.1262)			(0.0927)
Adj R^2	0.9832	0.9849	0.9855	0.9415	0.9587	0.9594

TABLE. V AR(P) BY "STANDARD DEVIATION" AND "VOLATILITY" SAMPLES

	Using "Standard Deviation" as sample			Using "Volatility" as sample		
	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)
S (t-1)	0.9911 ***	0.5104 ***	0.2968 ***	0.9649 ***	0.9130 ***	0.5965 ***
	(0.0154)	(0.0881)	(0.1088)	(0.0305)	(0.0707)	(0.1143)
S (t-2)		0.4855 ***	0.2956 **		0.0931	0.3120 **
		(0.0886)	(0.1132)		(0.0710)	(0.1340)
S (t-3)			0.4016 ***			0.1077
			(0.1295)			(0.0717)
Adj R^2	0.9745	0.9796	0.9799	0.9030	0.9556	0.9599

IV. Conclusion: In the report, different to the existed paper [4], we integrate Finite-Difference method and time series auto-regressive model to analyze two-section DFB self-sustaining-pulsation lasers periodic output stability. And we also classify the two level possible factors for Cross Section Data and Longitudinal Data.

Reference

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