Space Harmonic Distribution at Bragg Condition in Periodical Dielectric Waveguides

Nai-Hsiang Sun, Yu-Wei Liu, Ming-Yu Tsai, Li-Ti Kung, Jung-Sheng Chiang

Department of Electrical Engineering, I-Shou University,
1 Sec. 1, Syuecheng Rd., Dashu district
Kaohsiung, Taiwan 84001, R.O.C.
Phone: +886-7-6577711 ext. 6620, E-mail: snh@isu.edu.tw

***Abstract*-By Floquet-Bloch theory, the electric and magnetic fields are the summation of an infinite number of space harmonics, where the harmonics include forward and backward propagated harmonics. In this paper, the distribution of space harmonics are analyzed at the Bragg condition in grating structure of dielectric waveguides.**

I. Introduction

Contra-directional mode coupling in periodic dielectric waveguides is important in distributed Bragg reflector (DBR) lasers and distributed feedback (DFB) lasers. The geometry of the waveguide structure consists of a surface corrugation and two uniform dielectric waveguides. Figure 1 illustrates the basic geometry of the grating layer with a grating section. In this paper, we demonstrate the power distribution in the periodical waveguide structure. The Floquet-Bloch theory [1] and the Mahmoud-Beal's are applied to calculate the transmission and reflection spectra of the grating structure [2].

II. Problem Formulation

In the paper, the Floquet-Bloch theory (FBT) is used to analyze the periodical waveguide structure [1]. According to the Floquet-Bloch theory, the field pattern should consist of an infinite number of space harmonics. For the sake of simplicity it is assumed that the field is invariant with respect to *y*. Since the field is assumed to propagate in the z-direction, the solution of the Floquet-Bloch theory can be expressed as

$$E_y(x,z) = \sum_{n=-\infty}^{\infty} f_n^{(i)}(x)\cdot \exp(-jk_{zn}z) = E_{yf} + E_{yb} \qquad (1)$$

And

$$E_y^b(x,z) = \sum_{n=-\infty}^{\infty} f_n^{(i)}(x)\cdot \exp(jk_{zn}z) = E_{yf}^b + E_{yb}^b \qquad (2)$$

where

$$k_{zn} = \beta_n + j\alpha = (\beta_0 + nk) + j\alpha,\ \alpha < 0,\ K = \frac{2\pi}{\Lambda}$$

Here K is the grating wavenumber, *n* is the space harmonic order, and *i* represents the *i*th layer. The values k_{zn} is the complex propagation constant of the *n*th spatial harmonic. β is the propagation constant, α indicates attenuation constant. The real part of k_{zn} , β_n, is the phase constant of the nth space harmonic, and the imaginary part of k_{zn} ,α (α<0), is the attenuation constant due to the leakage of the guided-wave energy into the substrate and the superstrate regions. As expressed in (1) and (2), electric fields in the grating region consist of forward harmonics (E_{yf} and E_{yf}^b) and backward harmonics (E_{yb} and E_{yb}^b). Consider the vicinity of the first Bragg region. E_{yf} , E_{yb} , E_{yf}^b and E_{yb}^b can be written as

$$E_{yf} = \sum_{n=0}^{\infty} f_n(x)\cdot \exp(-jk_{kz}z) \qquad (3)$$

$$E_{yb} = \sum_{n=-\infty}^{-1} f_n(x)\cdot \exp(-jk_{kz}z) \qquad (4)$$

and

$$E_{yf}^b = \sum_{n=0}^{\infty} f_n(x)\cdot \exp(jk_{kz}z) \qquad (5)$$

$$E_{yb}^b = \sum_{n=-\infty}^{-1} f_n(x)\cdot \exp(jk_{kz}z)\,. \qquad (6)$$

Figure 1 shows that the fundamental mode (e_y) is launched at region I towards the grating. This produces the reflected and transmitted fields. As shown in Fig. 1, T_1 and R_1 are the transmission and reflection coefficients in the grating region, while t is the transmission coefficient in region II, and r is the reflection coefficient in region O. The coefficients T_1, R_1, and t, r are determined from the boundary conditions at the input and output planes of the grating.

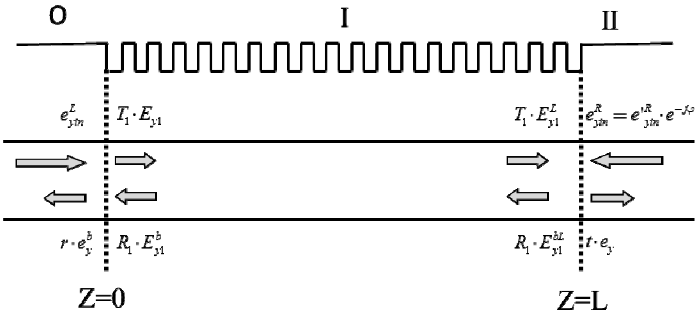

Fig. 1 The basic geometry of the two grating sections.

For the uniform plane wave, the boundary condition at the interference z=0 can be expressed as

$$e_{yin}^{L} + r \cdot e_y^b = T_1 \cdot \left(E_{y1f} + E_{y1b}\right) + R_1 \cdot \left(E_{y1f}^b + E_{y1b}^b\right) \quad (7)$$

$$h_{xin}^{L} - r \cdot h_x^b = T_1 \cdot \left(H_{x1f} + H_{x1b}\right) - R_1 \cdot \left(H_{x1f}^b + H_{x1b}^b\right) \quad (8)$$

In dielectric waveguide structure, the relationship between r, T_1, and R_1 is obtained by overlapping the boundary condition at non-grating and grating layer interfaces. The overlap equations are expressed as

$$\int e_{yin}^{L} \cdot h_x^* dx + r \cdot \int e_y^b \cdot h_x^* dx = T_1 \cdot \left(\int E_{y1f} \cdot h_x^* dx + \int E_{y1b} \cdot h_x^* dx\right) + R_1 \cdot \left(\int E_{y1f}^b \cdot h_x^* dx + \int E_{y1b}^b \cdot h_x^* dx\right) \quad (9)$$

$$\int h_{xin}^{L} \cdot e_y^* dx - r \cdot \int h_x^b \cdot e_y^* dx = T_1 \cdot \left(\int H_{x1f} \cdot e_y^* dx + \int H_{x1b} \cdot e_y^* dx\right) - R_1 \cdot \left(\int H_{x1f}^b \cdot e_y^* dx + \int H_{x1b}^b \cdot e_y^* dx\right) \quad (10)$$

Similar to Equations (9) and (10), we can express the boundary condition at the interference z=L. Finally, the coefficients r, t, T_1, and R_1 can be obtained.

The forward propagated power and the backward propagated power in the grating region can be expressed as follows.

$$P_f = \frac{1}{2}\mathrm{Re}\left[\int_{-\infty}^{\infty} (T_1 E_{y1f} + R_1 E_{y1b}^b) \cdot (T_1 H_{x1f} + R_1 H_{x1b}^b)^* dx\right] \quad (11)$$

$$P_b = \frac{1}{2}\mathrm{Re}\left[\int_{-\infty}^{\infty} (T_1 E_{y1b} + R_1 E_{y1f}^b) \cdot (T_1 H_{x1b} + R_1 H_{x1f}^b)^* dx\right] \quad (12)$$

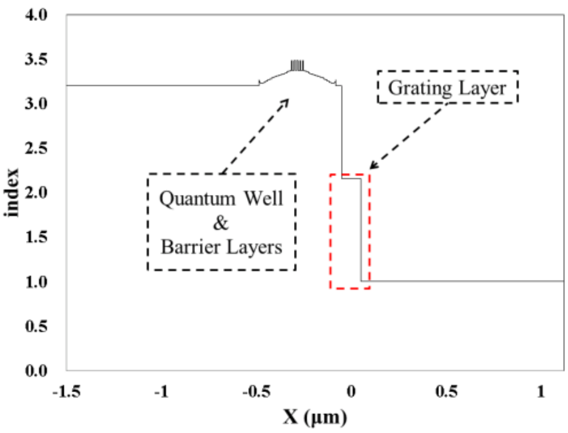

Fig. 2 The index profile of the periodical waveguide in the research.

III. Results

We consider the waveguide structure described in [3], where the free space wavelength is 1310 nm. Figure 2 shows the index profile of the periodical waveguide structure. We assume that there is no material loss in the layers.

According to the paramaters as discussed above, Fig. 3 shows the transmission and reflection spectra of the periodical dielectric waveguide, where the grating period (Λ) is 0.20276985 μm. The 3dB resonant wavelengths range from 1.3069 μm to 1.3119 μm with a bandwidth of 5×10^{-3} μm. The center resonant wavelength is 1.3094 μm.

Figure 4 shows the power distribution in the grating region with wavelength of 1315nm. The incident power is 1mW, while the transmission power and reflection power in non-grating regions are 9.9588 $\times10^{-1}$ mW and 4.4765 $\times10^{-3}$ mW, respectively. By using Eq. (13) and (14), the transmitted power and the reflected power in grating region are 9.9766 $\times10^{-3}$ mW and 7.2604 $\times10^{-2}$ mW. The differences between two regions are because we use the different method to calculate the power, whereas the relative error is small.

IV. Conclusion

In this paper, the reflection and the transmission efficiencies of periodical dielectric waveguide in a grating region are obtained by using the Mahmoud-Beal's method and the FBT.

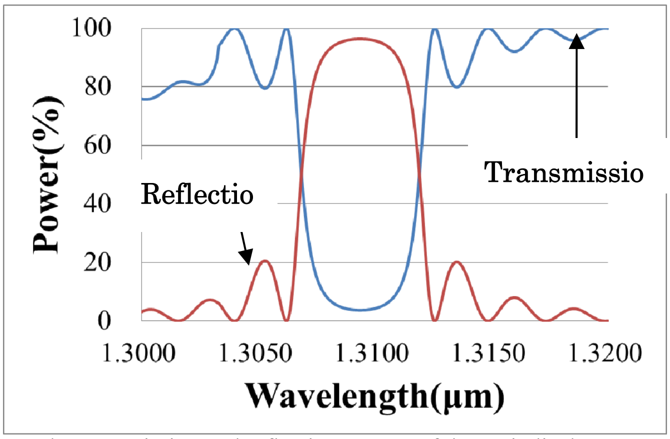

Fig. 3 The transmission and reflection spectra of the periodical waveguide with one grating period of 0.20276985 μm (Λ).

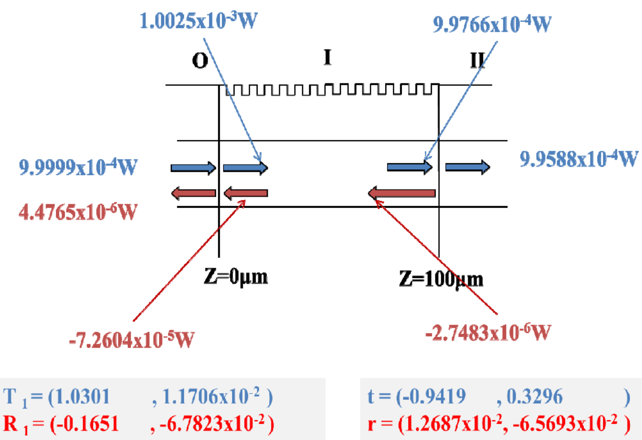

Fig. 4 The calculated power in the non-grating region and the grating region with wavelength of 1315nm.

References

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