Numerical Optimization of All-Optical Switching

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***Abstract*—A possibility to control an optical soliton by a much weaker second pulse that is scattered on the soliton attracted considerable attention recently. An important problem here is to quantify the small range of parameters at which the interaction takes place. This has been achived by using adiabatic ODEs for the soliton characteristics, which is much faster than a scan of the full propagation equations for all parameters in question.**

I. Introduction

We consider scattering of a low-intensity dispersive wave (DW) on an optical soliton mediated by cross-phase modulation in a nonlinear fiber. If the group velocity (GV) of the DW is close enough to the GV of the soliton then the DW is efficiently reflected and experiences frequency conversion [1], [2]. The soliton in turn acquires a permanent shift in frequency and time delay, moreover, it may experience an all-optical switching to a new state with a considerable gain (loss) in peak power [3]. The phenomenon has been observed in experiments, [4]–[6], and appears in many nonlinear wave systems [7].

Reflection of the DW can only be seen in a very small range of parameters. Since direct numerical simulations of soliton evolution are time consuming, the predicition of adequate initial parameter ranges is particularly useful. We shall quantify the DW reflection and find the parameter ranges in which the changes in soliton characteristics are most pronounced.

A suitable DW, even two orders of magnitude weaker than the soliton, can compress the latter to a single-cycle duration. The adequate DW parameters have been derived by using two simple coupled nonlinear Schrödinger equations (NLSE) of Manakov type. Our solution resides in results from soliton perturbation theory combined with quantum mechanical scattering theory for the DW. The predictions are tested against numerical solutions of the full generalized nonlinear Schrödinger equation (GNLSE).

II. An exemplary numerical simulation

Fig. 1(a) shows a typical GNLSE simulation in space-time domain. The monochromatic DW (A) approaches the initially stationary soliton (B) and, being reflected, yields an interference picture (C). The soliton is compressed and deflected (D). The reflected part of the DW is frequency shifted, as clearly seen in the frequency domain [Fig. 1(c)]. Like the DW, the soliton is frequency shifted during reflection [Fig. 1(b)].

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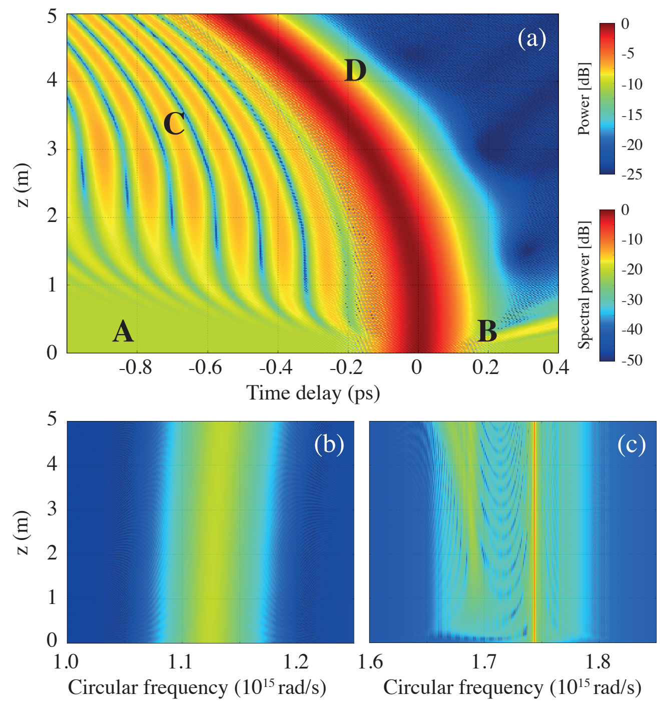

Fig. 1. An exemplary reflection of a DW from a soliton. (a) Normalized power in space-time representation. The DW is less intense than the soliton. The chosen frame of reference co-propagates with the unperturbed soliton. Normalized power in frequency domain for soliton (b) and DW (c).

Fig. 2(a,b) shows the group delay $\beta'(\omega)$, the group velocity dispersion $\beta''(\omega)$ (GVD), and indicates frequency shifts for the simulation shown in Fig. 1. Here the dispersion relation is encoded by $k = \beta(\omega)$. The carrier frequencies of soliton and DW (ω_a and $\omega_b + \Omega$ respectively) belong to opposite sides of the zero dispersion frequency at which $\beta''(\omega)$ vanishes, and are chosen such that $\beta'(\omega_a) = \beta'(\omega_b)$. Note that $\beta''(\omega_a) < 0$ and $\beta''(\omega_b) > 0$. Ω denotes the small initial DW frequency offset from ω_b.

III. Model

As can be seen in Fig. 1, the spectra of soliton and DW are neatly separated and remain so even after scattering. This observation suggests to describe the total electric field with two envelopes

$$E(z,t) = \mathrm{Re}\left[\psi_a(z,\tau)e^{i(\beta_a z-\omega_a t)} + \psi_b(z,\tau)e^{i(\beta_b z-\omega_b t)}\right]$$

where $\beta_{a,b} = \beta\left(\omega_{a,b}\right)$ and $\tau = t - \beta'_a z$ is introduced as the common retarded time. The complex envelopes ψ_a (for the

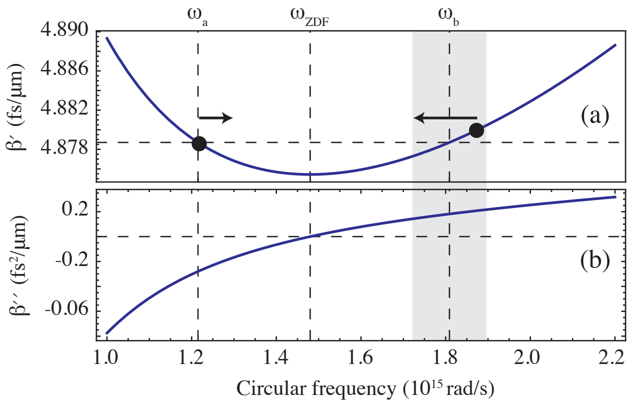

Fig. 2. A typical profile of (a) the group delay $\beta'(\omega)$ and (b) GVD $\beta''(\omega)$ that leads to the collision phenomenon shown in Fig. 1. Collision can only be realized for initial DW frequency offsets in a small interval (shaded grey) around the reference frequency of matching group velocity.

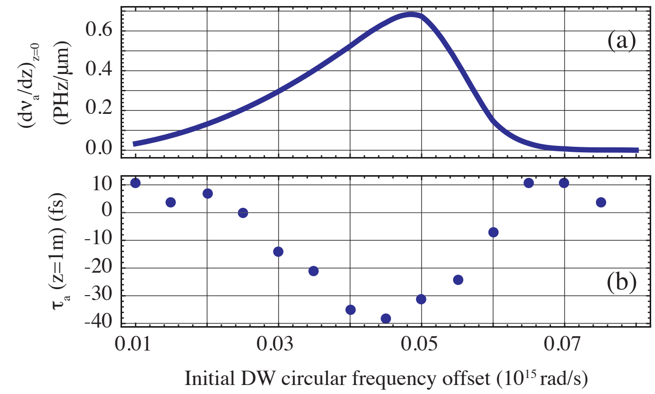

Fig. 3. Initial effect on the soliton as predicted by perturbation theory (a), and soliton deflection at propagation distance of 1m from full numerical simulation (b).

soliton) and ψ_b (for the DW) are governed by two coupled NLSEs

$$i\partial_z\psi_a - \frac{\beta_a''}{2}\partial_\tau^2\psi_a + \frac{n_{2a}}{c}\omega_a\left(|\psi_a|^2 + 2|\psi_b|^2\right)\psi_a = 0, \quad \text{(1a)}$$

$$i\partial_z\psi_b - \frac{\beta_b''}{2}\partial_\tau^2\psi_b + \frac{n_{2b}}{c}\omega_b\left(|\psi_b|^2 + 2|\psi_a|^2\right)\psi_b = 0. \quad \text{(1b)}$$

We reformulated this system in the following three steps which were suggested by observations in numerical simulations.

Firstly, the full solution of (1a) is approximated by a soliton the parameters of which are z-dependent, i.e.,

$$\psi_a(z,\tau) = \frac{1}{\sigma_a}\sqrt{\frac{|\beta_a''|c}{n_{2a}\omega_a}}\,\frac{\exp\left[-i\nu_a(\tau-\tau_a)+i\theta_a\right]}{\cosh\left[(\tau-\tau_a)/\sigma_a\right]} \qquad (2)$$

with duration $\sigma_a = \sigma_a(z)$, frequency offset $\nu_a = \nu_a(z)$, delay $\tau_a = \tau_a(z)$, and phase $\theta_a = \theta_a(z)$. In the absence of the DW

$$\frac{d\sigma_a}{dz} = \frac{d\nu_a}{dz} = 0, \quad \frac{d\tau_a}{dz} = \beta_a''\nu_a, \quad \frac{d\theta_a}{dz} = -\beta_a''\frac{\nu_a^2+\sigma_a^{-2}}{2}.$$

The latter equations have to be modified to account for interaction of the soliton with the DW.

Secondly, we insert $|\psi_a|^2$ from (2) in (1b). The latter is additionally linearized with respect to ψ_b, as the DW has much lower intensity compared to the soliton. Now (1b) describes the scattering problem of a plane wave at a squared hyperbolic secant barrier. It can be solved analytically for a static soliton barrier with the vanishing $\tau_a(z)$. To account for the soliton motion, a suitable Galilei transformation is applied to the standard scattering solution. The derived $|\psi_b|^2$ is inserted in (1a).

Thirdly, we revisit (1a) in which soliton perturbation theory [8] results in final evolution equations for the soliton parameters. Most importantly, we derive the following expression for the evolution of the solitons frequency

$$\frac{d\nu_a}{dz} = \frac{4\mu T}{\sigma_a L_a}\int_0^1 |F(\mathfrak{a},\mathfrak{b},\mathfrak{c},\zeta)|^2(2\zeta-1)\,d\zeta, \qquad (3)$$

where $\sigma_a(z) = \sigma_a(0)$, the dispersion length $L_a = \sigma_a^2/|\beta_a''|$ and μ is the DW power normalized by that of the soliton. Parameters that enter into the hypergeometric function F are

$$\mathfrak{a},\mathfrak{b} = \frac{1}{2} - i\varepsilon \pm is, \quad \mathfrak{c} = 1 - i\varepsilon, \quad s = \sqrt{4\frac{|\beta_a''|}{\beta_b''}\frac{\omega_b}{\omega_a}\frac{n_{2b}}{n_{2a}} - \frac{1}{4}},$$

$$\varepsilon = \left(\Omega - \frac{\beta_a''}{\beta_b''}\nu_a\right)\sigma_a, \qquad T = \frac{\sinh^2(\pi\varepsilon)}{\cosh^2(\pi s) + \sinh^2(\pi\varepsilon)}.$$

IV. Results

Equation (3) provides a simple and effective tool to estimate parameter ranges for initial parameters. We evaluated $d\nu_a/dz$ at $z = 0$ and for varying Ω, as depicted in Fig. 3(a) for a soliton with $\omega_a = 1.25$PHz and duration $\sigma_a = 55$fs. The curve shows the interval of interaction. At its maximum we expect to find the strongest initial effect on the soliton. Results are qualitatively valid for simulations with full GNLSE. For comparison, Fig. 3(b) shows τ_a at $z = 1$m. At the predicted optimal $\Omega \approx 0.05$PHz the absolute soliton deflection becomes maximal.

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