Discontinuous Galerkin Methods in Nano-Photonics

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***Abstract*—A review of the current status of Discontinuous Galerkin methods and their applications in nano-photonics is provided and future directions in methodic developments and applications are discussed.**

I. INTRODUCTION AND METHODOLOGY

Discontinuous Galerkin Time-Domain (DGTD) methods facilitate efficient computations of nano-photonic systems by combining the flexibility of finite element approaches with efficient explicit time-stepping capabilities [1], [2]. In fact, the former facilitates an accurate representation of complex geometries using unstructured meshes (see Fig. 1) and high-order basis functions, but does not impose any restriction on the latter. For instance, highly efficient low-storage Runge-Kutta schemes can straightforwardly be tailored for optimal performance [3] and sophisticated high-order multiple time-stepping algorithms may be used for dealing with the strongly non-uniform characteristics of the meshes of typical nano-photonic systems [4].

II. MATERIAL MODELS AND APPLICATIONS

In addition, time-domain solvers of the Maxwell equations with complex material properties (i.e., dispersive and/or nonlinear characteristics) require material models that are amenable to the technique of auxiliary differential equations. For instance, the linear properties of ordinary plasmonic materials such as gold or silver can be described by a Drude-model for the free electrons and one (or several) Lorentz-terms that account for interband transitions [2]. However, for small nano-particles such simple descriptions become inadequate and the nonlocal characteristics of the conduction electrons have to be taken into account. As a result, hydrodynamic extensions of the Drude model have been developed, albeit almost exclusively used together with frequency domain Maxwell solvers [5], [6]. Together with the group of L.M. Eng, we have recently implemented and studied a fully nonlinear version of the nonlocal hydrodynamic model [7]. In addition, we have developed an efficient perturbative approach to the second-order nonlinear response of this nonlocal and nonlinear hydrodynamic model that allows to discriminate the contributions of the electric and magnetic part of the Lorentz force [8], thus providing considerable further physical insight. By the same token, the electronic correlation effects in transition metals can be modeled by an extension of the Drude model and facilitates the efficient treatment of magneto-optical effects [9].

With this framework, we have analyzed optical scattering experiments [10] and electron energy loss spectroscopic studies of plasmonic nano-structures [12] and their coupling [11]. Furthermore, have investigated the modified fluorescence lifetimes of quantum emitters in the vicinity of nano-antennas [13].

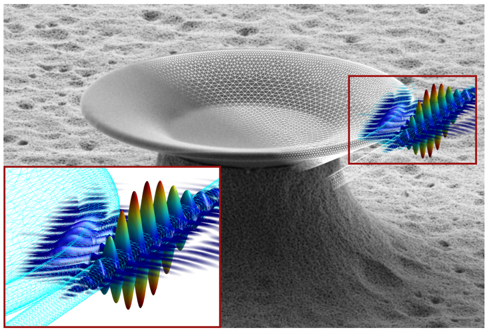

Fig. 1. Illustration of modeling realistic nano-photonic systems via DGTD: A nano-photonic resonator is mapped into an unstructured mesh of tetrahedra. In addition and a evanescently coupled fiber is introduced which carries an optical pulse to the resonantor and part of this pulse couples over to the resonator (see insets). The resonator has been adapted from [14]. Graphical Art courtesy of Richard Diehl.

III. CONCLUSIONS AND OUTLOOK

In summary, over the past years, the DGTD method has developed into a viable alternative to the venerable Finite-Difference Time-Domain method when it comes to time-domain simulations of nano-photonic systems.

Further progress is expected in many areas, both on a methodological level and with regards to applications. For instance, the high efficieny and accuracy of the DGTD method makes it an ideal tool for studying optical forces and the nonlinearities associated with them. Besides ordinary optical forces such as radiation pressure and the so-called gradient forces, this also includes the (numerically) more demanding fluctuation-induced forces such as Casimir and the Casimir-Polder forces. Similarly, while the presently available time-steppers are rather efficient, there still is considerable room for improvement, notably for systems with special characteristics. Finally, considering hybrid and/or multi-scale technques could prove to be highly rewarding in the not so distant future.

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