Time-Dependent Modulation Transfer Function and Quantum Efficiency for a N/P Photodiode

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***Abstract*– The performance of a N/P photodiode, in terms of spectral response and image contrast, is studied in a time-dependent mode thanks to numerical simulations. The minority carrier continuity equation has been solved with the Green's function approach in order to get a time-dependent carrier density. Afterward, the Modulation Transfer Function and the Quantum Efficiency have been calculated to estimate the image quality.**

***Keywords*– continuity equation, Green's function**

I. Introduction

Wide dynamic range image sensors use several integration times which can be very small. In [1], a CMOS image sensor allows an adaptive integration time which is between 150ns and 300ms. Besides, the theoretical highest frame rate of silicon image sensors is around 10^{11} frames per second [2], which leads to a very short integration time (10 picoseconds). The duration of generation and diffusion of charges becomes significant when the integration time is small. Then the steady state assumption in the differential equations can no longer be made. Generally the solution to the minority carrier continuity equation is obtained with exponential functions which do not depend on time [3][4]. Moreover the assumption of an infinite device is usually used. In this paper, we give an analytical solution to the continuity equation, according to time and for a finite size photodiode. The solution is obtained with the Green's function approach in a 3D rectangular coordinate system. The photonic current, which is created by the injected carriers, is time dependent. It has been used to calculate the Modulation Transfer Function (MTF) [4] and the Quantum Efficiency (QE) [5] in order to estimate the image quality according to time.

II. Time-Dependent Minority Carrier Density

A low-level injection of carriers is assumed in the P-region, then the continuity equation related to the excess

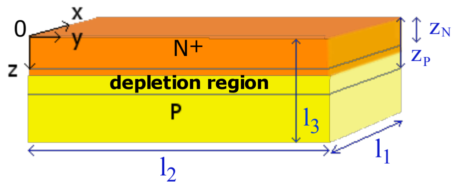

Fig. 1. N/P photodiode on a uniform substrate.

electron density n is [6]

$$\frac{\partial^2 n}{\partial x^2}+\frac{\partial^2 n}{\partial y^2}+\frac{\partial^2 n}{\partial z^2}-\frac{n}{L_n^2}+\frac{g}{D_n}=\frac{1}{D_n}\frac{\partial n}{\partial t}, \quad (1)$$

where L_n is the diffusion length, D_n is the diffusion coefficient, g is the generation function. Both of the functions g and n depend on the spatial coordinates (Fig. 1) and on time t. The following boundary conditions have been used

$$\begin{cases} n(x,y,z,t) &= 0 \quad \text{at all boundaries, } t>0 \\ n(x,y,z,0) &= 0 \quad \text{inside the photodiode.} \end{cases} \quad (2)$$

We simulated a point source of light in front of the photodiode for $t \geq 0$. Given the Beer-Lambert law, the generation function of carriers is

$$g(x,y,z)=M\delta\left(x-l_1/2,y-l_2/2\right)\alpha\exp(-\alpha z), \quad (3)$$

where l_1 and l_2 are the width and the length of the photodiode, α is the absorption coefficient, M is the incoming photon number, and δ is the dirac function. A detailed explanation of the resolution of the continuity equation in a P-region with the Green's function approach has been given by [7]. They only considered one spatial dimension, then the decrease of injected carriers with absorption depth is not taken into account. However the method of resolution can easily be adapted. For a 3D photodiode, the excess electron density n in the P-region is

$$\begin{aligned} n(x,y,z,t) &= \sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\sum_{p=1}^{\infty}\tfrac{8(-1)^{m+k}}{l_1 l_2(l_3-z_P)}\sin(\beta_m x)\sin(\gamma_k y) \\ &\times \tfrac{M\alpha\eta_p}{\alpha^2+\eta_p^2}\left(\exp(-\alpha z_P)-(-1)^p\exp\left(-\alpha l_3\right)\right) \\ &\times \tfrac{1-\exp\left(-D_n\left(\beta_m^2+\gamma_k^2+\eta_p^2+1/L_n^2\right)t\right)}{D_n\left(\beta_m^2+\gamma_k^2+\eta_p^2+1/L_n^2\right)}\sin(\eta_p(z-z_P)), \end{aligned} \quad (4)$$

where $\beta_m = \frac{(2m+1)\pi}{l_1}$, $\gamma_k = \frac{(2k+1)\pi}{l_2}$ and $\eta_p = \frac{p\pi}{l_3 - z_P}$. For numerical analysis, we need to truncate the infinite summations in (4). The minimum number of terms that we have to calculate depends on the parameters of the device, on time and on wavelength. Fig. 2 shows the diffusion of electrons when there is a point source of light with a wavelength of $0.9\mu m$. For a smaller wavelength, the generation of charges is closer to the top of the photodiode and the diffusion is less large.

A similar solution to the continuity equation can be found for the excess hole density p in the N-region with the Green's function method. For this region the velocity of surface recombination must be taken into account in the boundary conditions.

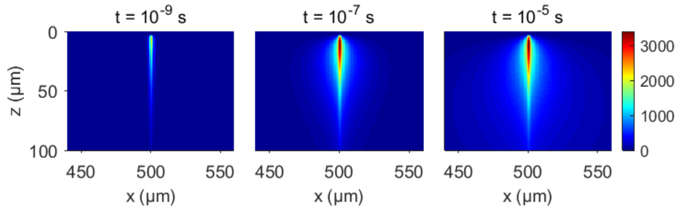

Fig. 2. Excess electron density according to the width x and the depth z, at $y = \frac{l_2}{2}$ and for a wavelength of $0.9\mu m$ ($l_1 = l_2 = 1000\mu m$ and $l_3 = 500\mu m$).

III. Simulation of MTF and QE

In order to assess the blur on images due to the diffusion of charges, the MTF has been calculated by taking the Fourier Transform of the current density (5). Here, the geometric MTF due to the integration of light over the photosensitive area is not studied. The first term in (5) represents the generation g of electron-hole pairs in the depletion region, the second term represents the diffusion of electrons from the P-region to the depletion region, and the last one, the diffusion of holes from the N-region to the depletion region [6].

$$J(x,y,t) = q\int_{z_N}^{z_P} g dz + qD_n \frac{\partial n}{\partial z}\Big|_{z=z_P} - qD_p \frac{\partial p}{\partial z}\Big|_{z=z_N} \quad (5)$$

QE is another parameter which is used to evaluate the sensitivity of a detector. It is defined as the ratio of the number of charges collected to the number of photons when there is a uniform incoming light. Hence (1) has been solved with the new generation of charges

$$g(x,y,z) = M\alpha \exp(-\alpha z). \quad (6)$$

Fig. 3 and Fig. 4 show the numerical simulations of MTF and QE, respectively. Because of the diffusion of charges, the MTF decreases with time during $1\mu s$ at $0.6\mu m$ wavelength and $10\mu s$ at $0.9\mu m$ wavelength. The number of generated charges increases with time and so does the QE.

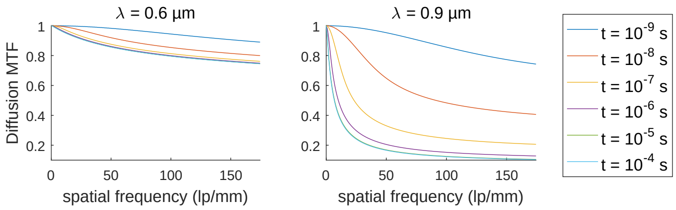

Fig. 3. Diffusion MTF according to spatial frequency (in line pairs per millimeter) for several wavelengths λ and times.

After $0.1ms$ the curve of QE does not change any more. We can see on the two last figures that the steady state is achieved at several times, depending on the wavelength.

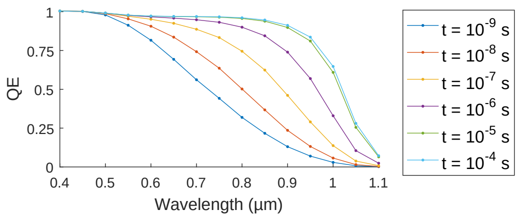

Fig. 4. QE according to wavelength at several times.

IV. Conclusion

The continuity equation has been solved with the Green's function so that we obtained a solution which depends on time for a finite photodiode. Afterward, the simulations showed that the diffusion MTF decreases and the QE increases with time until the steady state. These two parameters allowed an estimation of the sensitivity of the photodiode and of the contrast that we could have on an image during the transitional period. In addition, the simulations showed that the time needed to reach the steady state depends on the wavelength of the incoming light.

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