Self-mode-locking in Quantum Dot unidirectional ring lasers: model and simulations

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***Abstract*—We predicted the occurrence of spontaneous mode-locking phenomena in Quantum Dot based unidirectional ring laser via Risken-Nummedal-Graham-Haken instability of the continuous wave emission. Very interestingly for applications the resulting optical pulses have duration of few picoseconds with a THz repetition rate.**

I. Introduction

We theoretically studied coherent phenomena in the multi-mode dynamics of single section semiconductor unidirectional ring lasers with Quantum Dots (QD) active region similar to those considerend in [1], [2]. To this aim we extend the time domain traveling wave (TDTW) model presented in [3] to include the evolution equation of the medium polarization. Our simulations predict the occurrence of self-mode-locking (SML) in the system leading to ultrashort pulses ($\sim$ few picoseconds) with a THz repetition rate. The Linear Stability Analysis (LSA) of the continuos wave solutions (CW) is in good agreement with the numerics and it allows to establish an analogy between the observed CW instability and the well known Risken-Nummedal-Graham-Haken (RNGH) instability affecting the dynamics of two level lasers and consisting in the amplification of the Rabi frequency [4], [5]. While in the temporal domain this SML is very promising for the realization of compact lasing sources for time resolved measurement of e.g fast molecular dynamics and fast imaging system, in the frequency domain it correspond to a frequency comb made of narrow lines with almost equal amplitude and equal THz separation that let envisage applications in the field of long-range, high-capacity wireless communication based on combined THz photonics and THz electronics [6].

The results presented here well agree with recent theoretical and experimental evidences reported in literatures in the study of broadband Quantum Cascade Lasers that share with QD lasers a similar dynamical behaviour due to the almost symmetric optical response and the fast gain recovery time ($\sim$ few picoseconds) [7], [8].

II. The TDTW model

We consider a multilayer, multi-populations InAs QD laser emitting around 1.3 μm in a unidirectional ring configuration. Extending beyond the adiabatic elimination of the medium polarization the TDTW model presented in [3], the set of nonlinear PDEs that describe the dynamics of the slowly varying envelopes of the electric field E and of the microscopic polarization p_i for the i-th QD population, coupled with that of the occupation probabilities of the QD ground state ρ_i and of the Quantum Well wetting layer ρ_{WL} can be conveniently written in the following adimensional form:

$$
\begin{aligned}
\frac{\partial E}{\partial t}+\frac{\tau_d}{\tau_p}\frac{\partial E}{\partial z} &= \frac{\tau_d}{\tau_p}\left(-\frac{\alpha_{wg}L}{2}E - C\sum_{i=-N}^{N}\bar{G}_i p_i\right) && (1)\\
\frac{\partial p_i(z,t)}{\partial t} &= \left[(j\delta_i/\Gamma-1)p_i - D(2\rho_i-1)E\right] && (2)\\
\frac{\partial \rho_i(z,t)}{\partial t} &= \frac{\tau_d}{\tau_{sp}}\left[-\rho_i\gamma_e(1-\rho_{WL})\right.\\
&+ F\rho_{WL}\gamma_C(1-\rho_i)-\rho_i^2\\
&+ \left. H\,Re\left(E^* p_i\right)\right] && (3)\\
\frac{\partial \rho_{WL}(z,t)}{\partial t} &= \frac{\tau_d}{\tau_e^{WL}}\left\{\Lambda\tau_e^{WL}-\rho_{WL}\right.\\
&+ \sum_{i=-N}^{N}\left[-\bar{G}_i\rho_{WL}\gamma_C^{WL}(1-\rho_i)\right.\\
&+ \left.\left.\frac{\bar{G}_i}{F}\rho_i\gamma_e^{WL}(1-\rho_{WL})\right]\right\} && (4)
\end{aligned}
$$

The reference frequency ω_0 coincides with one of the "cold cavity modes" $\omega_n = 2\pi n v_g/L$, $n \in Z$ (v_g = group velocity of the light in the medium) and with the gain peak; τ_d and τ_p are the dipole dephasing time and the photon lifetime respectively, τ_{sp} is the spontaneous carriers decay time, τ_e and τ_C are the escape and capture time from and to the ground state respectively and τ_e^{WL} is the carriers nonradiative decay time in the wetting layer; $\alpha_{wg}/2$ represents the waveguide losses per unit length, $\bar{G}_i$ is the probability that a QD belongs to the i-th population, δ_i and Γ are the central frequency and the FWHM of the homogeneously broadened gain line associated to the i-th population, Λ is the probability per unit time that an injected electron is captured into a wetting level of a QD layer. The constant coefficients C, D, F, H depend on the QD laser geometry and the active material. Time is scaled on τ_d while the longitudinal coordinate is scaled on the cavity length L. Finally E obeys periodic boundary conditions. For all the QD material parameters we refer to those used in [3]. In the following we report some important results obtained in the study of the transition from the single frequency, or CW, emission to the multi longitudinal mode dynamics using the biased current I as control parameter.

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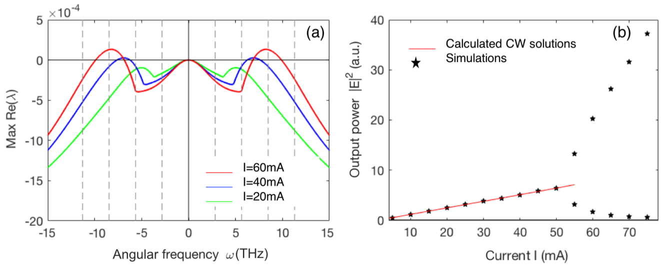

Fig. 1. We consider L=200 μm and 3 active populations with characteristic interband transition energies of $E_{GS} = \hbar\omega_0 = 0.9879eV$, E_{GS}+4.2 meV, E_{GS}−4.2meV that correspond to the frequencies 0 THz, 6.45 THz −6.45 THz with our choice of the reference frequency. The other parameters are as in [3]. (a) LSA of the CW solutions for different values of biased current I. Dashed lines correspond to the cavity modes ω_n. (b) Bifurcation diagram of the CW solutions obtained using the pump current I as control parameter. The symbols denote the maxima and minima values of the simulated output power, while the red line denotes the calculated values of the stable CW solutions.

A. CW solutions and Linear Stability Analysis

We calculate the CW solutions of Eqs. (2)-(4) in the form: $E = \overline{E}e^{j(\delta\omega/\Gamma\, t - \delta k\, Lz)}$, $p_i = \overline{p_i}e^{j(\delta\omega/\Gamma\, t-\delta k\, Lz)}$, $\rho_i = \overline{\rho_i}$, $\rho_{WL} = \overline{\rho_{WL}}$ where the overbars denote constant quantities and $\delta k = \delta\omega/v_g$, by numerical solution of the nonlinear algebraic system obtained by inserting the previous expressions in Eqs. (2)-(4).

The study of the CW stability against spatio-temporal perturbations leads to a set of linear equations for time evolution of the perturbation Fourier components with wave vectors $k_n = 2\pi n/L$. For each k_n, the real part of the Lyapunov exponents λ represent thus the growing rate of the perturbation amplitude. For different values of I we plot for example in Fig. 1a the real part of the largest eigenvalue λ versus $\omega = k_n v_g$ treating as a continuous variable. Dashed lines represent the cold cavity modes ω_n. We observe that the CW solution emitted at the lasing threshold becomes unstable for $I \geq 60$ mA where the parametric gain overcomes the losses (positive value of the largest real part of λ) for the perturbation wave vectors $k_{\pm 3}$. In analogy with the RNGH instability affecting the CW solutions in multimode two level lasers we find that the peaks of the positive parametric gain that trigger the CW instability are almost resonant with the Rabi frequencies associated with each population and estimated following the standard procedure [5] (not reported here for brevity).

III. Numerical simulations: Self-mode-locking

The prediction of the LSA are confirmed by the numerical integration of Eqs. (2)-(4) as shown by the bifurcation diagram in Fig. 1b. As in the RNGH instability for the two level laser (resonant case) [4], self-mode-locking leads to the emission of optical pulses with picosecond duration and repetition rate of the order of the typical Rabi oscillations in the THz range (see Fig. 2). In agrement with the LSA the spacing between spectral lines in Fig. 2b corresponds to around three times the Free Spectral Range ($\sim$ 8500 THz).

The numerics also reveal that the increase of the number of active populations and consequently of the inhomogeneous broadening (that represents an additional dispersion mechanism) reduces the interval of biased current where self-pulsing is found (see for example Fig. 3). Finally, in presence of carriers grating generated by standing wave patterns in bidirectional QD ring lasers or in the more conventional Fabry-Perot configurations, our preliminary results show that the CW instability threshold decreases down to few percents above the lasing threshold and that there are sizeable intervals in the bias current where phenomena of phase-locking still occur although they are not associated with emission of optical pulses [9].

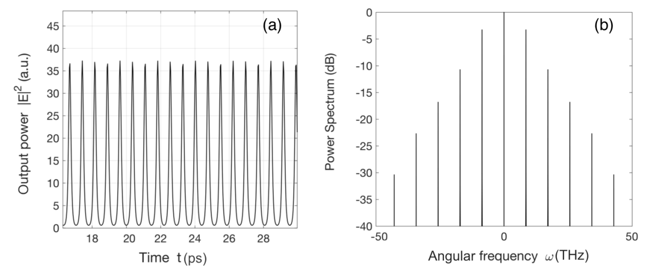

Fig. 2. (a) Temporal evolution of the output power and (b) the associated power spectrum in a self-pulsing configuration corresponding to $I = 75$ mA. The other parameters are as in Fig. 1.

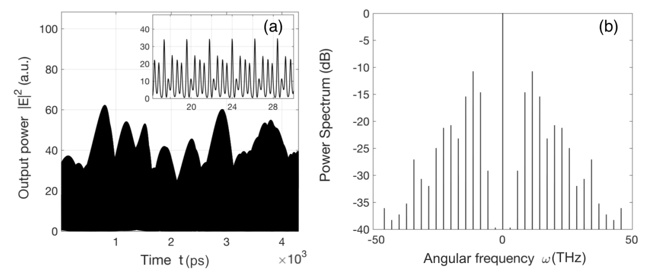

Fig. 3. We consider 5 active populations while the other parameters are as in Fig. 1. (a) Temporal evolution of the output power and (b) the associated power spectrum in a self-pulsing configuration corresponding to $I = 75$ mA. The inset represents a zoom of the time trace in panel (a) for comparison with Fig. 2a.

IV. Conclusion

We studied the dynamics of multimode unidirectional QD ring lasers and we predicted the occurrence of SML beyond the RNGH instability threshold of the CW solutions that leads to the emission of ultrashort pulses with THz repetition rate.

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