Ultrashort solitons and their control in the regime of event horizons in nonlinear dispersive optical media

U. Bandelow*, Sh. Amiranashvili*, S. Pickartz*
*Weierstrass Institute (WIAS), Mohrenstrasse 39, 10117 Berlin, Germany. Email: Uwe.Bandelow@wias-berlin.de

***Abstract*—Several key concepts for the description of nonlinear pulse propagation in dispersive optical media will be introduced. Among those are envelope models, short-pulse equations, as well as our generalized approach [1]. Effects like limitations for ultrashort optical pulses [2], cusp formation [3], and solitons that mimic event horizons for smaller optical waves will be presented. Moreover, it will be demonstrated, both numerically and more efficiently by a new analytic theory [4], that small optical waves can be used to control such solitons [5], [6].**

I. Ultrashort Pulses

Pulses in nonlinear optical fibers are typically described by the nonlinear Schrödinger equation (NLSE). In the case of short pulses with wide spectra, additional effects, as higher-order dispersion and Raman-scattering have to be taken into account by a generalized nonlinear Schrödinger equations (GNLSE). The GNLSE provides a better description: too short pulses become wide in the frequency domain, such they are inevitably affected by frequency components evolving in the region of positive group velocity dispersion (GVD). These pulses are then either *destroyed*, because solitons in focusing materials require negative GVD, or they *increase in duration* due to Cherenkov radiation. An alternative approach to the description of few-cycle pulses is to employ non-envelope equations, that are designed to describe non-envelope pulses in particularl and directly calculate their electric field. In this way, they provide more information compared to envelope models, in particular they remove the arbitrariness of the carrier-envelope relation. Such equations are loosely referred to as short pulse equations (SPE) [7], which, to some surprise, have proven to be integrable shortly after [8]. With decreasing pulse duration its solitary solutions become increasingly sharp and finally a cusp singularity develops at the top of the soliton [9]. Neither the limiting cusp soliton nor further singular solutions of SPE, which are even shorter than the cusp solution, are acceptable from the physical point of view. Therefore, the SPE sets a natural limit to the duration of optical solitons. At half maximum the shortest duration contains approximately 1.5 optical cycles of the pulse field.

That appearance of the cusp singularity seems to be a universal mathematical feature that governs the shortest soliton both in optical systems, and even beyond optics, for the following reasons. First, the same singular behavior of ultrashort solitons has been found in alternative complex SPE's, both for first- and second order propagation equations [10]. Second, it was found that if the standard NLSE is generalized by accounting for amplitude-dependent group velocity, the corresponding solitary solution readily shows cusp formation for too short pulses [11]. Third, optical cusp solitons closely resemble singular solutions that appear in seemingly different physical systems, e.g., cuspons and peakons that appear in Camassa Holm equation for shallow water waves. Last, but not least, the appearance of all singular solutions in the above examples can be described by the same mathematical structure. Namely, the solitary solution ϕ of the SPE in question appears as a homoclinic trajectory of the reduced dynamical system

$$(\phi')^2 + 2U(\phi, P) = \text{const},$$

where the effective potential $U(\phi,P)$ depends on the choice of the SPE and, most important, on the pulse duration P. As to the shortest duration, $U(\phi,P)$ transforms in an infinite potential well (red line in Fig. 1a). The corresponding solution for ϕ is still continuous but has different values of ϕ' for the incoming and outgoing branches of the homoclinic trajectory, which provides a cusp solution. Remarkably, the fundamental soliton of the simplest NLSE with the same duration still reproduces the shape of the limiting soliton reasonably well, except the cusp at the top (Fig. 1b). However, the shortest duration can be obtained only from the singular solution of the SPE.

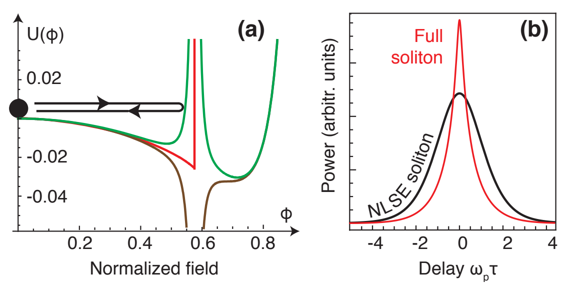

Fig. 1. (a) Effective potentials calculated for different durations of optical solitons. Green: regular potential that yields a typical ultrashort soliton. Red: limiting case (infinite potential wall) resulting in an unphysical cusp. Brown: too short durations lead to singularities, solitons do not longer exist [10]. (b) Shortest soliton (red) for Drude dispersion, versus standard fundamental soliton (black line), see [2].

Cusp existence was also confirmed in a general setting by direct numerical solution of Maxwell equations for several dispersion laws [2], using a non-envelope bidirectional nonlinear wave equation with cubic nonlinearity. A typical close-to-cusp-but-still-regular solution is shown in Fig. 1b, similar to analytical solutions resulting from the SPEs. A special feature is the decay of the spectral power of cusp solutions inversely proportional to the fourth degree of frequency [3].

II. Pulse Interaction

An optical pulse that propagates along a fiber with Kerr nonlinearity, creates a localized nonlinear perturbation δn of the refractive index. For instance, a 3-cycle (half-maximum) soliton in fused silica at $1.55\,\mu$m provides $\delta n \approx 10^{-4}$. A

978-1-5386-5599-3/18/$31.00 ©2018 IEEE

co-propagating pulse would usually pass the perturbation unchanged, under favorable conditions it is scattered however [12]. A suitable group velocity matched pump wave may even be perfectly reflected, thereby undergoing a pronounced frequency change [13]. The reflected wave propagates in the same direction as the soliton but with a different velocity due to frequency shift, as schematically shown in Fig. 2.

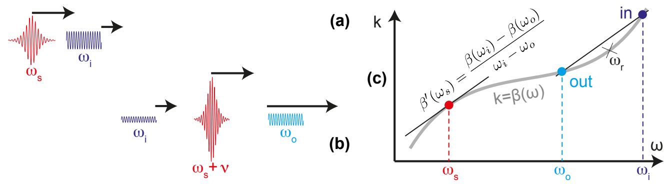

Fig. 2. (a) fiber soliton (red) and dispersive wave (DW) packet (dark blue) effectively interact with each other if they co-propagate with only slightly different velocities. (b) a new frequency-shifted DW (light blue) appears after reflection. The frequency change $\omega_i \mapsto \omega_o$ indicates energy transmission, the soliton gets pumped and propagates with changed peak power and shifted frequency $\omega_s \mapsto \omega_s + \nu$. (c) Both ω_i and ω_o are close to the velocity matched frequency ω_r with $\beta'(\omega_s) = \beta'(\omega_r)$.

A frequency down-shift of the scattered wave $\omega_i \mapsto \omega_o$ indicates an energy exchange: the pump feeds the soliton, which increases in peak power and also experiences a frequency shift $\omega_s \mapsto \omega_s + \nu$. Thus a soliton can be manipulated by a carefully chosen pump wave, which should be a low-amplitude group-velocity matched continuous dispersive wave (DW). For instance, the soliton can be switched on and off [12], trapped [14], and even used to mimic event horizons [15]. We present analytic theory [4] of interactions like the one shown in Fig. 3, quantify optimal pulse parameters [5], and demonstrate how optical event horizons can be used to compensate the Raman effect in a stable manner (Fig. 4, [6]).

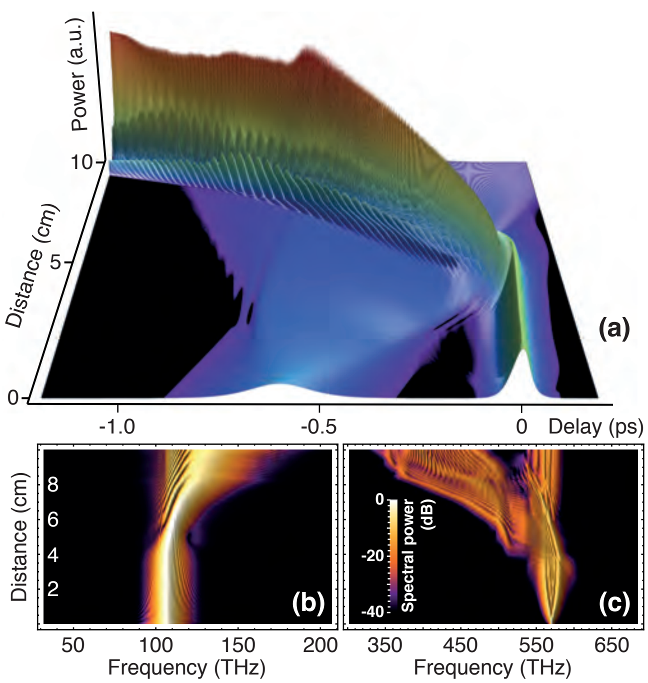

Fig. 3. (a) an exemplary scattering of the DW (left pulse) at a soliton (right pulse) in the space-time domain. (b) spectral density for the soliton. (c) spectral density for the DW. See [4] for details.

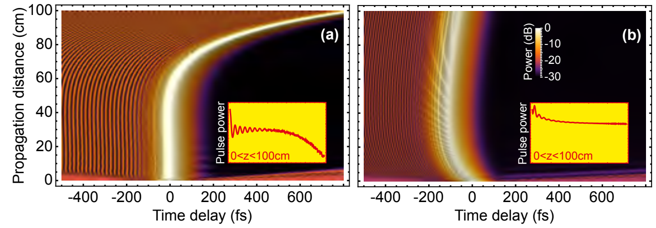

Fig. 4. Energy density plot of the incoming/scattered DW (interference pattern at the beating frequency $\omega_i - \omega_o$ on the left side of each panel) and a fundamental soliton with initially zero delay (soliton parameters are identical to that in Fig. 3). SSFS compensation by DW scattering may be (a) unstable or (b) stable. The adiabatic approach to such soliton-DW interactions makes it possible to address the stability problem without tedious numerical calculations of pulse propagation. See [6] for details.

Acknowledgment

U.B. acknowledges support by the German Research Foundation in the framework of the Collaborative Research Center 787 Semiconductor Nanophotonics under project B5. Sh.A. acknowledges support of the German Research Foundation under Project 389251150.

References

[1] S. Amiranashvili, U. Bandelow, and A. Mielke, "Calculation of ultrashort pulse propagation based on rational approximations for medium dispersion," *Opt. Quantum Electron.*, vol. 44, no. 3, pp. 241–246, 2012.

[2] S. Amiranashvili, U. Bandelow, and N. Akhmediev, "Few-cycle optical solitary waves in nonlinear dispersive media," *Phys. Rev. A*, vol. 87, no. 1, p. 013805, 2013.

[3] ——, "Spectral properties of limiting solitons in optical fibers," *Optics Express*, vol. 22, no. 24, pp. 30 251–30 256, 2014.

[4] S. Pickartz, U. Bandelow, and S. Amiranashvili, "Adiabatic theory of solitons fed by dispersive waves," *Phys. Rev. A*, vol. 94, p. 033811, 2016.

[5] ——, "Efficient all-optical control of solitons," *Opt Quant Electron*, vol. 48, pp. 503:1–7, 2016.

[6] ——, "Asymptotically stable compensation of soliton self-frequency shift," *Opt. Lett.*, vol. 42, no. 7, pp. 1416–1419, 2017.

[7] T. Schäfer and G. E. Wayne, "Propagation of ultra-short optical pulses in cubic nonlinear media," *Physica D*, vol. 196, no. 1-2, pp. 90–105, 2004.

[8] A. Sakovich and S. Sakovich, "The short pulse equation is integrable," *Journal of the Physical Society of Japan*, vol. 74, no. 1, pp. 239–241, Sep. 2005.

[9] ——, "Solitary wave solutions of the short pulse equation," *Journ. Phys. A*, vol. 39, no. 22, pp. L361–L367, 2006.

[10] S. Amiranashvili, A. G. Vladimirov, and U. Bandelow, "Solitary wave solutions for few-cycle optical pulses," *Phys. Rev. A*, vol. 77, p. 063821, 2008.

[11] S. Amiranashvili, U. Bandelow, and N. Akhmediev, "Dispersion of nonlinear group velocity determines shortest envelope solitons," *Phys. Rev. A*, vol. 84, no. 4, p. 043834, 2011.

[12] A. Demircan, S. Amiranashvili, and G. Steinmeyer, "Controlling light by light with an optical event horizon," *Phys. Rev. Lett.*, vol. 106, no. 16, p. 163901, April 2011.

[13] D. V. Skryabin and A. V. Yulin, "Theory of generation of new frequencies by mixing of solitons and dispersive waves in optical fibers," *Phys. Rev. E*, vol. 72, no. 1, p. 016619, 2005.

[14] A. V. Gorbach and D. V. Skryabin, "Light trapping in gravity-like potentials and expansion of supercontinuum spectra in photonic-crystal fibres," *Nature Photonics*, vol. 1, no. 11, pp. 653–657, 2007.

[15] T. G. Philbin, C. Kuklewicz, S. Robertson, S. Hill, F. Konig, and U. Leonhardt, "Fiber-optical analog of the event horizon," *Science*, vol. 319, no. 5868, pp. 1367–1370, 2008.