Dynamics of two mutually coupled semiconductor lasers in low coupling regions

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Abstract— **We study a system of two mutually delay-coupled semiconductor lasers for integration in a Photonic Integrated Circuit. This system is described by single mode rate equations, which are a system of delay differential equations with one fixed delay. A comprehensive frequency analysis presented, which predicts stable symmetric and symmetry-broken one-colour, periodic, quasi-periodic and undamped relaxation oscillation states for low coupling regions. These states are confirmed with the obtained bifurcation diagrams. The obtained frequencies are benchmarked against the experimental outcomes.**

I. Introduction

We investigate a system of two mutually delay-coupled semiconductor lasers, in a face to face configuration in a Photonic Integrated Circuit (PIC). This system has very interesting laser dynamics and is important in the creation of advanced modulation formats within a PIC [1] by using regions of stability where no dynamics are seen. Fig. 1 shows the microscopic image of the devices under study. The lasers are connected with a variable optical attenuator (VOA) section, and are coherently coupled via their optical fields, where the time delay τ arises from the finite propagation time of the light from one laser to the other. This system can be described by the Lang-Kobayashi rate equations, which are a system of delayed differential equations (DDEs) with one fixed delay. Yanchuk et al. [2] studied this model in the limit of small and zero delay and predicted one-colour symmetric states. Later Erzgräber et al. [3] studied the bifurcations of one-colour states for large delay. Moreover, for zero delay, stable symmetric and symmetry-broken one-colour and two-colour states have been predicted by Clerkin et. al. [1]. We have recently investigated this system for finite delays, where we observed that the symmetric and symmetry-broken, one-colour and two-colour stables states continue to exist at high coupling regions [4]. These results are consistent with the bifurcation diagrams calculated using the continuation software DDE-BIFTOOL [5].

Here we investigate the frequencies and bifurcation of the coupled lasers in a low coupling regions. As the separation between lasers d, is fixed in a given integrated device, the impact of time delay can only be studied by comparing multiple devices. Hence, the devices under investigation have

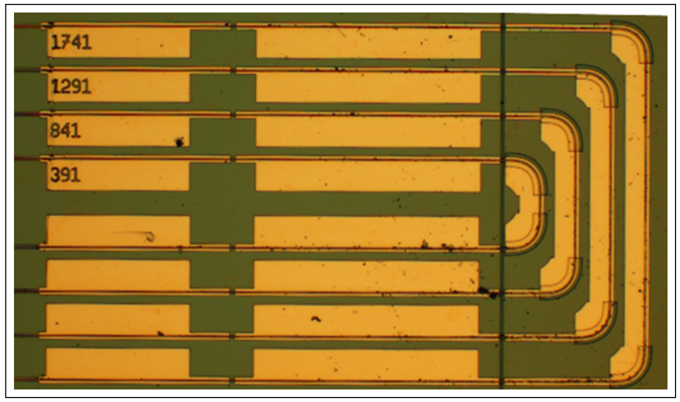

Fig. 1. The microscope image of the devices under study, with varying VOA lengths. The U-shap VOAs are chosen to avoid substrate coupling in the VOA section.

a VOA lengths in the range 391-1741 nm (shown in Fig. 1), which is equivalent to time delays between 0.5-2.

II. Rate equation model

The rate equations in the absence of detuning and in the reference frame of ω_0, are given by [3]

$$\frac{\mathrm{d}E_{1,2}(t)}{\mathrm{d}t} = (1+i\alpha)N_{1,2}(t)E_1(t)+\kappa e^{-iC_p}E_{2,1}(t-\tau), \quad (1)$$

$$T\frac{\mathrm{d}N_{1,2}(t)}{\mathrm{d}t} = P - N_{1,2}(t) - (1+2N_{1,2}(t))|E_1(t)|^2, \quad (2)$$

Here E_1 and E_2 are the normalized complex slowly varying envelope of the optical fields and N_1 and N_2, are the normalized inversions for laser 1 and laser 2, respectively. α, T and P are parameters of the laser. $\alpha = 2.6$ is the linewidth enhancement factor, $P = 0.23$ is the pumping parameter, and $T = 392$ is the normalized carrier lifetime. The time t is normalized to the photon lifetime τ_p which is estimated to be around 7.7 ps [4]. The main parameters in this study are time delay $\tau(= nd/c)$, coupling phase C_p and coupling strength κ. The coupling strength κ can be associated with the fraction of photons coupled from one laser into the other, and varies between 0 and 1. In practice κ can be controlled by the applied voltage to the variable optical attenuator (VOA) section. The coupling phase C_p changes with the central frequency as $C_p = \omega_0\tau \mod 2\pi$.

The system of DDEs in Eqs. 1 and 2 are solved numerically for a given τ, κ and C_p. The calculated optical field frequencies indicate the existence of symmetric and symmetry-broken one-colour, periodic, quasi-periodic and undamped relaxation oscillation states for different values of parameters.

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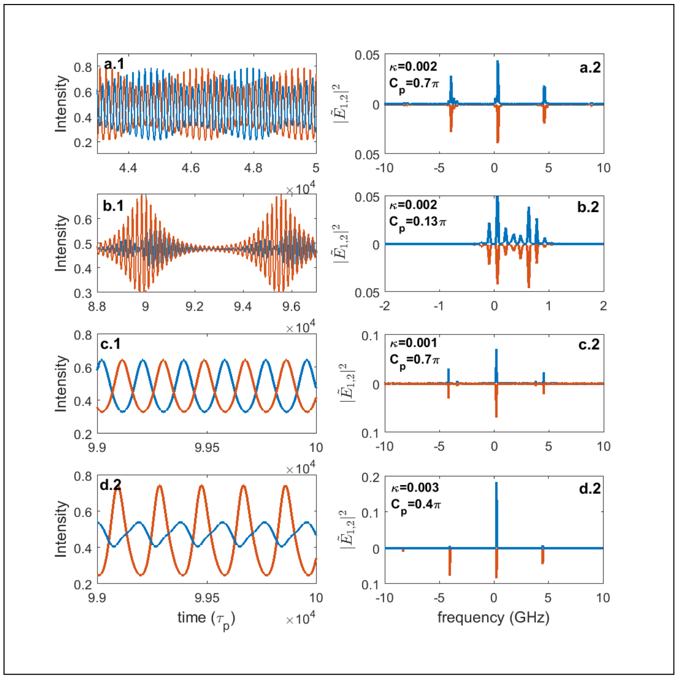

Fig. 2. Time traces (left) and frequency spectra (right) of lasers 1, for $\tau = 0.5$ and several values of coupling strength κ, and coupling phase C_p. Figs (a) and (b) show examples of symmetric and symmetry-broken quasi-periodic states, respectively. Figs (c) and (d) present examples of symmetric and symmetry-broken undamped relaxation oscillation states, respectively.

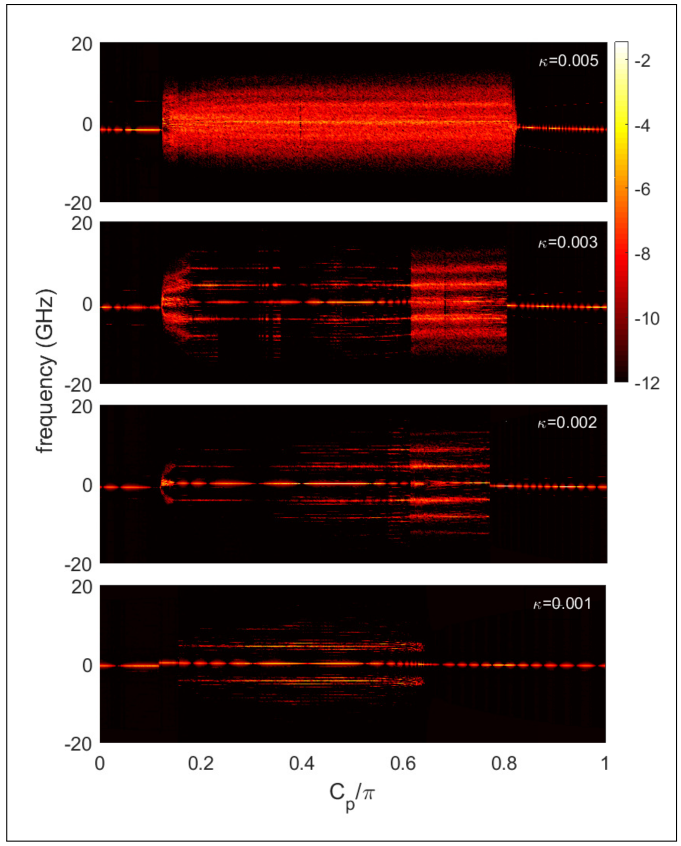

Fig. 3. Frequencies of laser 1 versus phase, C_p, for $\tau = 0.5$ and several values of coupling $\kappa = 0.001$, 0.002, 0.003 and 0.005.

III. Results and Discussion

In order to solve Eqs. 1-2 numerically we first separate the real and imaginary parts of E_1 and E_2. Then the system of 6 DDEs is solved numerically using Matlab code based on an explicit Runge-Kutta scheme. For given parameters C_p, κ and τ we calculate the optical fields $E_{1,2}$ and inversions $N_{1,2}$ for laser 1 and 2, respectively. The left panel in Fig. 2 show the time traces of optical fields for $\tau = 0.5$, and selected values of C_p and κ. The Fourier transforms of the time trace of the slowly varying optical fields ($\tilde{E}_1$ and $\tilde{E}_2$) allows us to study the optical frequency output of the lasers, as shown in the right panel of Figs. 2. Figs. (a.1) and (a.2) show stable symmetric quasi-harmonic states which indicates that both lasers operate at a common frequencies with the same amplitude. In Figs. (b.1) and (b.2) the intensity of lasers have the same frequencies with different amplitudes, which indicate symmetry-broken quasi-harmonic states. Similarly, Figs. c and d shows examples of symmetric (c) and symmetry-broken (d) undamped relaxation oscillation states. In order to get a better picture of the behaviour of the system, we study the optical frequency spectrum of the lasers for constant coupling κ and varying phase C_p. Fig. 3 shows the variation of frequency versus phase, calculated numerically for $\tau = 0.5$ and constant coupling phases: $\kappa = 0.001$, 0.002, 0.003 and 0.005. For $\kappa = 0.001$, consistent with the bifurcation diagram [4], we observe symmetric in-phase and anti-phase one-colour states for $C_p < 0.15\pi$ and $C_p > 0.65\pi$. For the middle range of the phase C_p, a symmetry-broken multi-colour region was observed. As seen in Fig. 3 for the higher coupling ($kappa = 0.002 - 0.003$), we observe symmetric multi-colour region for C_p between 0.6 and 0.8, which means the shrinkage in the anti-phase one-colour region. For the coupling higher that 0.05 (and lower than 0.5) we observe chaotic behavior. However, for higher coupling values we see symmetry-broken two-colour states. [4].

IV. Conclusion

The presented results confirms the existence of the stable solutions, including symmetric in-phase and anti-phase one-colour as well as, symmetric and symmetry-broken quasi-periodic and undamped relaxation oscillations states, for the system in the low coupling regions. We are currently benchmarking our result with the experiments.

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