Implementation of Rectified Ring Resonator Model into Coupled Mode Theory through Appropriate Inner Boundary Conditions

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***Abstract*- We describe a numerical method to simulate the modal behavior of microring resonators based on the extension of Coupled Mode Theory (CMT). Our approach uses straightened, equivalent waveguide model with appropriate inner boundary conditions and allows the numerical combination of various structural elements.**

I. INTRODUCTION

Semiconductor-based tunable light sources are key devices for digital coherent systems. To enhance the effectiveness and transmission capacity, wider tuning range as well as narrower linewidth is required, while maintaining low power consumption. Microring resonators are promising reflective optical elements for these novel signal light sources due to their customizable reflectance spectra and compact size.

The ability to use simple mathematical models to simulate microrings is a key capability behind their success in practical applications. There are previously formulated methods available to model them such as Finite Difference Time Domain (FDTD) [1], semi-analytical [2], conformal transformation [3] and modified Coupled Mode Theory (CMT) [4] by defining integral expression for coupling coefficient. In this paper we show a new approach based on the extension of CMT with appropriate inner boundary conditions. It allows to handle microrings as straightened waveguides and enables the easier numerical combination with various gratings, waveguides in the algebraic eigenvalue problem solved by Finite Difference Method.

II. THEORY AND NUMERICAL MODEL

The lasing mode of DFB or DBR-based tunable lasers is usually calculated and analyzed by using CMT [5]. A resonator mode is decomposed to forward and backward propagating waves: $E(z) = E_f(z)e^{-i\beta z} + E_b(z)e^{i\beta z}$, where amplitudes E_f and E_b can only have slow variation compared to the exponentials. Substituting this assumption into 1D Helmholtz equation and applying equidistant discretization for N points leads to an algebraic eigenvalue problem:

$$\begin{bmatrix} \frac{i}{\Delta z}\boldsymbol{D} + \beta\boldsymbol{I} & -\boldsymbol{K} \\ -\boldsymbol{K} & \frac{-i}{\Delta z}\boldsymbol{D} + \beta\boldsymbol{I} \end{bmatrix} \begin{bmatrix} E_{f,1} \\ \vdots \\ \hline E_{b,1} \\ \vdots \end{bmatrix} = k_0 \begin{bmatrix} \boldsymbol{N} & 0 \\ 0 & \boldsymbol{N} \end{bmatrix} \begin{bmatrix} E_{f,1} \\ \vdots \\ \hline E_{b,1} \\ \vdots \end{bmatrix} \quad (1)$$

Here **D** stands for the symmetric difference operator (matrix), the diagonal **N** and **K** matrices for the material parameters, **I** is the unit matrix and k_0 is the complex wavenumber. The enforced boundary conditions (facet reflectivities) are:

$$E_{b,N} = r_r E_{f,n} \text{ and } E_{f,1} = r_f E_{b,1} \quad (2)$$

This algebraic problem can be solved by Finite Difference Method. When a microring is put at a facet the constrain on reflectivity becomes invalid and must be substituted with a more complicated one. We derive these constrains for rectified models of grating-free and grating-integrated microrings.

A. Extension of CMT for grating-free microring

We assume a two-armed grating-free micro-ring reflector located on the right side of the laser cavity (Fig. 1.)

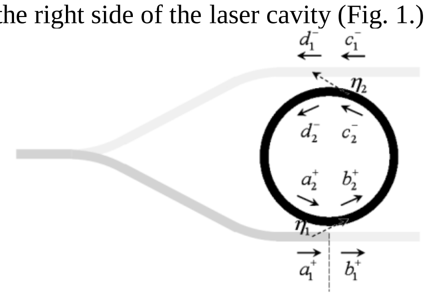

Fig. 1. Geometry of a two-armed grating-free microring

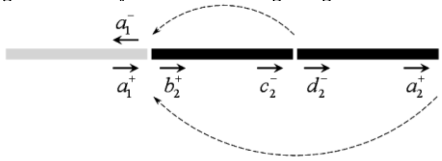

Fig. 2. Rectified model of a two-armed grating-free microring (the upper waveguide section can be omitted from the analysis)

The lengths and effective indices of the two waveguide sections must be identical except for the coupling coefficients η_1 and η_2. In this case, it is enough to analyze the forward traveling wave in the bottom waveguide (a_1^+), which is partially reflected via the top arm (d_1^-). The forward traveling wave in the top arm could be reflected via the bottom arm similarly. Since the exact path of the traveling wave is not important, d_1^- can be substituted with a_1^-, which allows the rectification of the relevant section of the arrangement.

Inner boundary conditions are enforced at the entrance and exit points of the straightened microring:

$$\begin{bmatrix} b_1^+ \\ b_2^+ \end{bmatrix} = \begin{bmatrix} \tau_1 & j\eta_1 \\ j\eta_1 & \tau_1 \end{bmatrix} \begin{bmatrix} a_1^+ \\ a_2^+ \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} a_1^- \\ d_2^- \end{bmatrix} = \begin{bmatrix} \tau_2 & j\eta_2 \\ j\eta_2 & \tau_2 \end{bmatrix} \begin{bmatrix} c_1^- \\ c_2^- \end{bmatrix} = \begin{bmatrix} j\eta_2 c_2^- \\ \tau_2 c_2^- \end{bmatrix} \quad (4)$$

We extended the algebraic eigenproblem (Eq. (1)) with the inner points of the microring for the counterclockwise traveling wave, and the independent boundary variables a_1^+, a_2^+, c_2^-. Dependent variables can be eliminated from the matrix exploiting Eqs.(3) and (4). Note that the clockwise traveling wave is not formulated explicitly, because it would correspond to the otherwise analogous upper waveguide section.

B. Extension of CMT for grating-loaded microring

Ring with integrated grating has one waveguide bus and can be modeled as straightened waveguide with grating at the center.

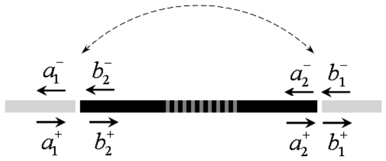

Fig. 3. Rectified model of a grating integrated microring

After discretization of inner points of ring, linear relationships among the amplitudes give inner boundary conditions. The ring coupling coefficient is relabeled as η.

$$\begin{bmatrix} b_1^+ \\ b_2^+ \end{bmatrix} = \begin{bmatrix} \tau & j\eta \\ j\eta & \tau \end{bmatrix} \begin{bmatrix} a_1^+ \\ a_2^+ \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} a_1^- \\ a_2^- \end{bmatrix} = \begin{bmatrix} \tau & j\eta \\ j\eta & \tau \end{bmatrix} \begin{bmatrix} b_1^- \\ b_2^- \end{bmatrix} = \begin{bmatrix} j\eta b_2^- \\ \tau b_2^- \end{bmatrix} \quad (6)$$

Eq.(5) is irrelevant for the eigenvalue problem, as it just gives the outcoupled amplitude. Therefore, here the algebraic eigenproblem (Eq.(1)) should be extended with the inner variables of the microring section, and the independent a_1^+, a_2^+, b_2^-. $b_1^- = 0$, and the other dependent variables are eliminated using Eqs.(5) and (6) together with the boundary condition for the other side of the straight grating.

III. RESULTS, VALIDATION

The method was implemented in MATLAB and the cold cavity reflectivity characteristic of microrings were successfully reproduced. Here simple grating-free cavity (L_w=600µm) bounded by a high-reflectivity broadband mirror (R_f=64%), and grating-free microring ($\eta_1 = \eta_2 = \eta$, Perimeter: L_p=300µm) is analyzed. This structure was optimized for 1550nm main lasing wavelength.

A. No internal and bending losses in the cavity

Eq.(1) was solved for the rectified model (L= L_w+ L_p=900µm) using outer boundary condition at front facet (Eq.(2)) and inner ones (Eqs.(3-4)) at ring section. The calculated cold cavity longitudinal modes ($I(z) = |E(z)|^2$) shows that the intensity in the ring is reduced to half for η = 1, as only the forward traveling wave is coupled into the ring (Fig. 4). The intensity is zero in the second half of the microring (the gray background represents the straightened ring). The ring coupled intensity (Γ_{ax}: mode confinement in ring) and the loss difference between the 1st and 0th modes increase for smaller η, indicating higher finesse value. The loss of the central mode and cavity mode spacing decreases for smaller η, because the effective cavity length gets longer due to more roundtrips.

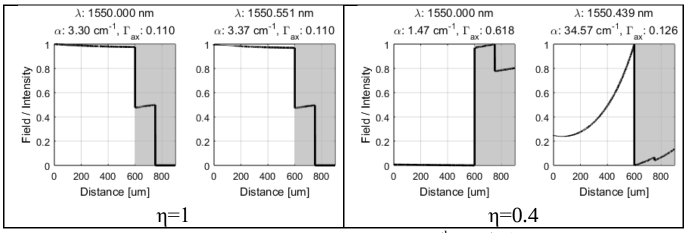

Fig. 4. Cold cavity longitudinal modes (0th, +1st) for various coupling coefficients (η)

B. Lossy microring configuration

The bending loss can be substituted with absorption type internal loss in the rectified model, incorporated into the imaginary part of the complex refractive index. Here $\alpha_{int}(z)$ can contain any type of internal loss along the longitudinal cavity.

$$\alpha_{int} = -\frac{2\, ln(\alpha_{90})}{\pi R}, \; n(z) = n'(z) + i\,\frac{\lambda \alpha_{int}(z)}{4\pi} \quad (7)$$

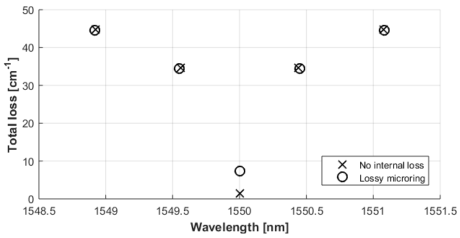

Fig. 5. Cavity mode data for lossless and lossy ring configuration (cold cavity)

When bending loss was assumed (η = 0.4, $\alpha_{int} = 10cm^{-1}$), the loss of 0th mode has increased the largest amount, because its intensity profile had largest overlap with the ring (Fig 5.).

IV. CONCLUSION

We described and validated an effective numerical method for longitudinal mode analysis of microring resonators. The inner points of ring are handled as straight waveguide in CMT applying appropriate boundary conditions. This approach allows any combination of waveguide, grating and microring structures in CMT, and results quick simulation of longitudinal mode profiles with their resonance wavelengths and modal losses.

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