Modeling phase noise in high-power photodetectors

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***Abstract*—We describe the simulation model that we use to calculate the impulse response and phase noise in a modified uni-traveling carrier (MUTC) photodetector using the drift-diffusion equations while avoiding computationally expensive Monte Carlo simulations.**

Phase noise in photodetectors is a critical limiting factor in many RF-photonic applications [1]. Quinlan *et al.* [2] experimentally showed that while there is a significant reduction in the phase noise as the pulse duration decreases, this decrease in the phase noise ceases once the optical pulse duration becomes small compared to the duration of the electrical pulse that emerges from the photodetector. Sun *et al.* [3] were able to reproduce these experimental results using Monte Carlo simulations. However, they did not take advantage of the fact that the distribution of electrons in any time slot is expected to be Poissonian, which simplifies both the calculations and the physical interpretation of the results. In our approach, we use the drift-diffusion equations, combined with the observation that the arrival of electrons in any time interval is Poisson-distributed, to calculate the phase noise. This approach takes minutes on a desktop computer, as opposed to the many hours on a computer cluster that the Monte Carlo approach requires. The results that we present in Fig. 4 previously appeared in Jamali *et al.* [4]. Here, we present previously unpublished details on our computational approach.

Our starting point is to use the drift-diffusion equations [5] to calculate electron density n, hole density p, and potential distribution ψ across the photodetector. Since explicit methods are intrinsically unstable and require an unreasonably small time step to yield physical results with the diffusion coefficients, it is important to use a fully implicit method [6] when discretizing the drift-diffusion equations for numerical computation. We used the implicit Euler method to discretize the equations in time t. We used second-order finite differences to discretize the spatial dimension x. This discretization was previously described by Hu *et al.* [5].

Figure 1 shows the numerical mesh that we used for the finite difference spatial discretization of one-dimensional (1-D) drift-diffusion equations.

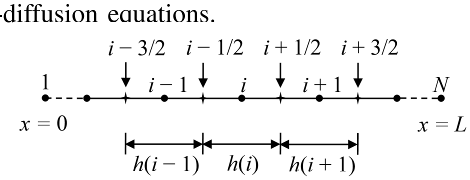

Fig. 1. Numerical mesh used for the finite difference spatial discretization of the 1-D drift-diffusion equations.

We approximate the electric field at the half-integer points in the mesh as

$$E_{i+1/2} = -\left(\frac{\psi_{i+1} - \psi_i}{h_i}\right), \tag{1}$$

where ψ_i is the potential at mesh-point i, and we approximate $\partial p/\partial x$ and $\partial n/\partial x$ at the half-integer points as

$$\begin{aligned} \left.\frac{\partial p}{\partial x}\right|_{i+1/2} &= \left(\frac{p_{i+1} - p_i}{h_i}\right), \\ \left.\frac{\partial n}{\partial x}\right|_{i+1/2} &= \left(\frac{n_{i+1} - n_i}{h_i}\right), \end{aligned} \tag{2}$$

where h_i is distance between the points $i-1/2$ and $i+1/2$. We calculate the currents at the half-integer points by discretizing the drift-diffusion equations to obtain

$$\begin{aligned} \mathbf{J}_{p,i+1/2} &= qp_{i+1/2}\mathbf{v}_{p,i+1/2}(\mathbf{E}) - qD_{p,i+1/2}\left(\frac{p_{i+1} - p_i}{h_i}\right), \\ \mathbf{J}_{n,i+1/2} &= qn_{i+1/2}\mathbf{v}_{n,i+1/2}(\mathbf{E}) + qD_{n,i+1/2}\left(\frac{n_{i+1} - n_i}{h_i}\right), \end{aligned} \tag{3}$$

where $p_{i+1/2} = (p_{i+1} + p_i)/2$, $n_{i+1/2} = (n_{i+1} + n_i)/2$, $D_{n,i+1/2}$ and $D_{p,i+1/2}$ are the electron and hole diffusion coefficients at the point $i + 1/2$, and $\mathbf{v}_{n,i+1/2}(\mathbf{E})$ and $\mathbf{v}_{p,i+1/2}(\mathbf{E})$ are the electron and hole drift velocities as a function of the electric field at the point $i + 1/2$. Finally, we approximate

$$\begin{aligned} \frac{\partial \mathbf{J}_{p,i}}{\partial x} &= \frac{\mathbf{J}_{p,i+1/2} - \mathbf{J}_{p,i-1/2}}{[(h_i + h_{i-1})/2]}, \\ \frac{\partial \mathbf{J}_{n,i}}{\partial x} &= \frac{\mathbf{J}_{n,i+1/2} - \mathbf{J}_{n,i-1/2}}{[(h_i + h_{i-1})/2]}. \end{aligned} \tag{4}$$

The total output current is the sum of the electron, hole, and displacement currents and is given by

$$\mathbf{J}_{\text{total}} = \mathbf{J}_n + \mathbf{J}_p + \epsilon\frac{\partial \mathbf{E}}{\partial t}. \tag{5}$$

To calculate the impulse response, we first calculate the steady state output current. We then perturb the generation rate by ΔG_{opt} and calculate the impulse response due to the perturbed ΔG_{opt}. We use

$$\Delta G_{\text{opt}} = rG_{\text{opt}}\,\text{sech}\left(\frac{t}{\tau}\right), \tag{6}$$

where G_{opt} is the optical generation rate, r is the perturbation coefficient, sech(t) is the hyperbolic secant function, t is time, and τ is the impulse width. We set $\tau = 10$ fs, which we verified produces reliable results for the impulse response for times that are larger than 40 fs. We use $r = 10^{-1}$, which we have verified is sufficiently small that no nonlinear effects occur, while it is large enough to avoid roundoff errors. The normalized impulse response $h(t)$ is then given by

$$h(t) = \frac{\Delta I_{\text{out}}(t)}{\int_0^\infty \Delta I_{\text{out}}(t)dt}, \tag{7}$$

so that $\int_0^\infty h(t)dt = 1$, where $\Delta I_{\text{out}}(t)$ is the change in the output current due to the impulse.

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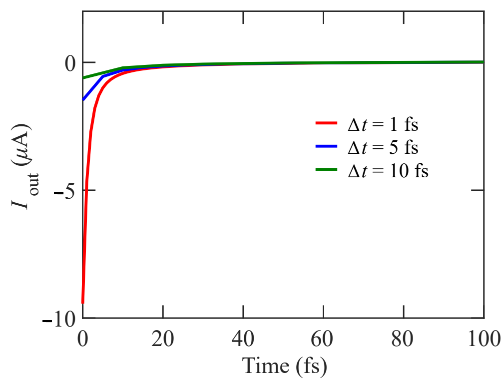

Fig. 2. Output current (I_{out}) of the MUTC photodetector as a function of time for different time meshes (Δt).

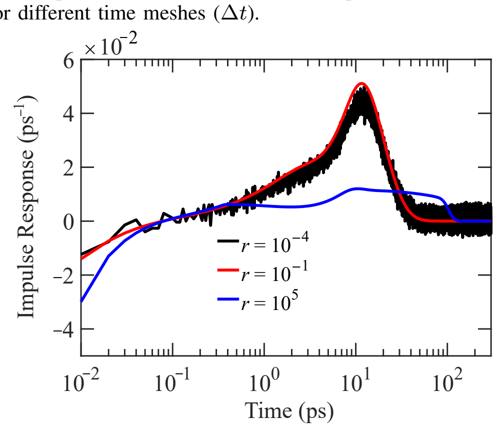

Fig. 3. Normalized impulse response of the MUTC photodetector as a function of time for different values of r.

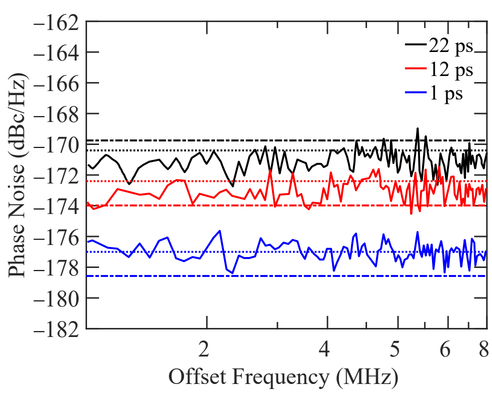

Fig. 4. Phase noise of the MUTC photodetector as a function of offset frequency from the fifth harmonic at 10 GHz for three different optical pulse widths. Dot-dashed lines are experimental results of Quinlan *et al.* [2]; solid lines are Monte Carlo simulation results of Sun *et al.* [3]; dotted lines are our simulation results.

We note that $h(t)$ as defined here includes the combined effect of a finite optical pulse duration and the electrical response to the optical pulse. This definition is consistent with Refs. [2] and [3]. In order to verify that our results are independent of the choice of τ and r, we ran numerical tests in which we allowed these quantities to vary. In Fig. 2, we show output current (I_{out}) of a modified uni-traveling carrier (MUTC) photodetector [7] as a function of time t for different time meshes (Δt). The results are almost identical for $t > 20$ fs, indicating that the frequency dependence will be reliable up to frequencies of 50 THz, which is far beyond the limit of 10–50 GHz at which experiments indicate that the device can no longer respond. In Fig. 3, we show how the calculated impulse response varies for the MUTC device as r varies. When $r = 10^{-4}$, computational errors degrade the impulse response, leading to rapid fluctuations. When $r = 10^5$, nonlinearity becomes important, and the impulse response is distorted. For $10^{-3} < r < 10^4$, the impulse response is almost identical to the impulse response when $r = 10^{-1}$, which we have shown.

We define $h(t)$ as the response of the photodetector to a finite-duration optical pulse, as in [2], [3]. Because the arrival of electrons in any interval Δt varies from shot-to-shot and is Poisson-distributed, the number of electrons in that interval has a variance equal to $h(t)N_{\text{tot}}\Delta t$, from which we calculate the phase noise. We find that when the optical pulse duration τ is less than about 500 fs, $h(t)$ tends to a finite limit $h_e(t)$, which has a duration on the order of 10 ps. In this limit, we obtain

$$\left\langle \Phi_m^2 \right\rangle = \frac{1}{N_{\text{tot}}} \frac{\int_0^{T_R} h_e(t) \sin^2\left[2\pi m(t-t_c)/T_R\right] \mathrm{d}t}{\left\{\int_0^{T_R} h_e(t) \cos\left[2\pi m(t-t_c)/T_R\right] \mathrm{d}t\right\}^2}, \tag{8}$$

where $\left\langle \Phi_m^2 \right\rangle$ is the mean-square phase fluctuation, N_{tot} is the total number of electrons in the photocurrent, m is the harmonic number, and t_c is the central time of the photocurrent. Hence, in the limit of short optical pulse durations, we find that $\langle \Phi_m^2 \rangle$ tends to a non-zero constant. Figure 4 shows the experimental and calculated phase noise of the MUTC photodetector as a function of offset frequency from the fifth harmonic at 10 GHz for three different optical pulse widths. As Fig. 4 demonstrates, we obtain good agreement with both experimental and Monte Carlo simulation results.

In conclusion, we used the drift-diffusion equations combined with the observation that the arrival of electrons in any time interval is Poisson-distributed to calculate the phase noise in an MUTC photodetector avoiding time-consuming Monte Carlo simulations. We used the implicit Euler method to discretize the equations in time t. We used second-order finite differences to discretize the spatial dimension x. We obtain good agreement with both experimental and Monte Carlo simulation results.

References

[1] V. J. Urick, Keith J. Williams, and Jason D. McKinney, *Fundamentals of Microwave Photonics* (Wiley, New Jersey, 2015).

[2] F. Quinlan, T. M. Fortier, H. Jiang, A. Hati, C. Nelson, Y. Fu, J. C. Campbell, and S. A. Diddams, “Exploiting shot noise correlations in the photodetection of ultrashort optical pulse trains,” Nat. Photonics **7**, 290–293 (2013).

[3] W. Sun, F. Quinlan, T. M. Fortier, J. D. Deschenes, Y. Fu, Scott A. Diddams, and J. C. Campbell, “Broadband noise limit in the photodetection of ultralow jitter optical pulses,” Phys. Rev. Lett. **113**, 203901 (2014).

[4] S. E. Jamali Mahabadi, S. Wang, T. F. Carruthers, C. R. Menyuk, F. J. Quinlan, M. N. Hutchinson, J. D. McKinney, and K. J. Williams, “Calculation of the impulse response and phase noise of a high-current photodetector using the drift-diffusion equations,” Opt. Express, **27**, 3717–3730 (2019).

[5] Y. Hu, B. S. Marks, C. R. Menyuk, V. J. Urick, and K. J. Williams, “Modeling sources of nonlinearity in a simple PIN photodetector,” J. Lightw. Technol. **32**, 3710–3720 (2014).

[6] S. Selberherr, *Analysis and Simulation of Semiconductor Devices* (Springer, New York, 1984).

[7] Z. Li, H. Pan, H. Chen, A. Beling, and J. C. Campbell, “High-saturation-current modified uni-traveling-carrier photodiode with cliff layer,” IEEE J. Quantum Electron. **46**, 626–632 (2010).