Control of Solitons in the regime of event horizons in nonlinear dispersive optical media

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Abstract—**We describe the propagation of nonlinear pulses in dispersive optical media on base of our generalized approach [1]. It is known, that intense pulses, such as solitons, can mimic event horizons for smaller optical waves. We prove that such strong pulses can be dramatically influenced in the course of nonlinear interaction with the proper dispersive waves. Moreover, it will be demonstrated, both numerically and more efficiently by a new analytic theory [2], that small optical waves can be used to control such solitons [3], [4]. In particular, the typical pulse degradation caused by Raman-scattering can be completely compensated by these means [4], which is supported by recent experiments [5].**

I. INTRODUCTION

Optical data transmission at high bitrates along fibers requires (ultra-) short pulses that propagate in a stable manner to establish faultless transfer of information. These pulses inevitably suffer from various detrimental effects during propagation, such as fiber dispersion, losses, and nonlinearity, to name the fundamental effects amongst them. If fiber dispersion and nonlinearity compensate we talk about solitons, which are stably propagating intense pulses. This can be analytically described by the integrable nonlinear Schrödinger equation (NLSE) [6]. In the case of short pulses with wide spectra, additional effects, such as higher-order dispersion and Raman-scattering become increasingly important, which cannot be compensated in simple ways, if at all. In modeling the propagation of nonlinear pulses, these higher order effects have to be taken into account by generalized nonlinear Schrödinger equations (GNLSE) [1], [7], or by various short pulse equations (SPE) [8]. The latter are designed to describe non-envelope pulses and directly calculate their electric field. To some surprise, many SPE's have proven to be integrable [9].

The most general form of propagation equation for an optical pulse envelope $\psi(z,\tau)$ where z is measured along the fiber and the delay $\tau = t - z/v_g$ is the following:

$$i\partial_z \psi + \hat{\mathscr{D}}\left[i\partial_\tau\right]\psi +$$
$$\gamma\left(1 + \frac{i}{\omega_0}\partial_\tau\right)\psi\int_{-\infty}^{\infty} R(\tau')|\psi(z,\tau-\tau')|^2 d\tau' = 0. \quad (1)$$

It includes general dispersion by the operator $\hat{\mathscr{D}}[i\partial_\tau]$, Kerr-nonlinearity (coefficient γ), and self-steepening, involved by the τ-derivative of the last term, as well as Raman scattering, described by the Raman response function $R(\tau')$ in the last term. Note, that this equation is nonlocal in time in general, which reflects the delayed response of the medium and causality in a natural way. As a consequence, special algorithms are required for its numerical solution [10]. We will present both numerical calculations and theory based on Eq. (1).

II. PULSE INTERACTION

An optical pulse that propagates along a fiber with Kerr nonlinearity, creates a localized nonlinear perturbation δn of the refractive index. For instance, a 3-cycle (half-maximum) soliton in fused silica at $1.55\,\mu$m provides $\delta n \approx 10^{-4}$. A co-propagating pump pulse would usually pass the perturbation unchanged, under favorable conditions it is scattered however [11]. A suitable group velocity matched pump wave may even be perfectly reflected, thereby undergoing a pronounced frequency change [12]. The reflected wave propagates in the same direction as the soliton but with a different velocity due to frequency shift, as schematically shown in Fig. 1.
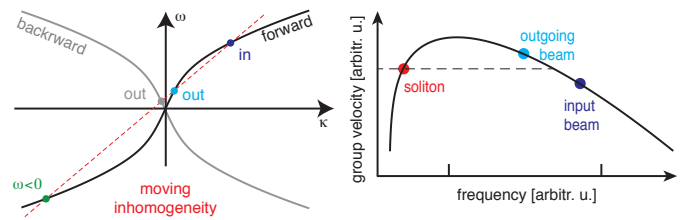

Fig. 1. A fiber soliton (red) and a dispersive wave (DW) packet (dark blue) effectively interact with each other if they co-propagate with only slightly different velocities. This is shown by the group velocity dispersion profile on the right. A new frequency-shifted DW (light blue) appears after reflection, its spectral position is given by the resonances (cross of red soliton line with the black dispersion curve) on the left. Interestingly, there is an additional resonance (green) for the forward wave at a negative frequency.

A frequency down-shift $\omega_i \mapsto \omega_o$ of the scattered wave indicates an energy exchange: the pump feeds the soliton, which increases in peak power and also experiences a frequency shift $\omega_s \mapsto \omega_s + v$. Thus, a soliton can be manipulated by a carefully chosen pump wave, e.g., by a low-amplitude group-velocity matched continuous dispersive wave (DW). For instance, the soliton can be switched on and off [11], trapped [13], and even used to mimic event horizons [14].
We present an analytic theory [2] of interactions like the one schematically shown in Fig. 1. This theory results in a coupled set of ordinary differential equations for soliton parameters and thereby reduces dramatically the effort compared to the numerical solution of the partial differential equation (1). Moreover, our theory allows even qualitative understanding of this interaction in simple terms, it allows to quantify optimal pulse parameters [3], and even to estimate the stability of the chosen control schemes.

III. RESULTS

Under suitable conditions solitons can mimic event horizons for dispersive waves by total reflection [14]. In the course of the mutual nonlinear interaction, the soliton is remarkably

affected by the (nonlinear) interaction with the much smaller dispersive wave. In consequence the soliton trajectory changes, together with its spectrum, such that it can become transparent later. This behaviour strongly supports the idea of using dispersive waves for control of soliton trajectories and shapes.

We will further show, that the inclusion of the *self-steepening* term in Eq. (1) is crucial for the description of this interaction, because it counts properly for the mutual cross-phase modulation [15].
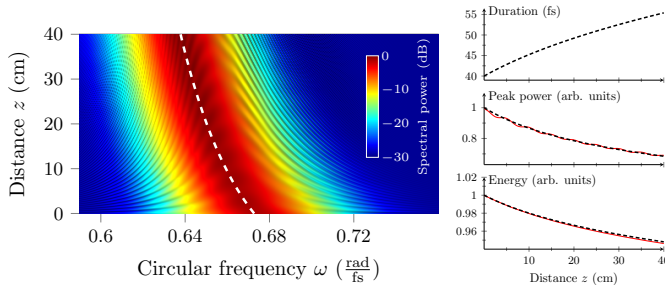

Fig. 2. Soliton, suffering Raman SSFS during propagation. Left: Evolution of the spectral power of the soliton, showing the red shift. Right: Evolution of characteristic soliton parameters. Top: temporal width (increases). Middle: peak power (decreases), as well as the energy (bottom). See [4] for details.

Furthermore, our theory nicely reproduces former results on *Raman scattering* [16], expressed even in simpler terms [4], see Fig. 2. We further demonstrate, that the interaction of solitons with DW's under conditions close to optical event horizons can be used to counteract the Raman effect, see Fig. 3.
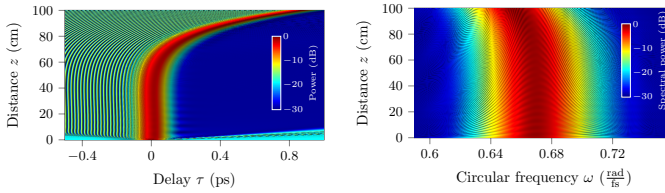

Fig. 3. (Unstable) compensation of SSFS by a DW, scatterd at a soliton. Left: Space-time trajectory, showing stabilization up to 60 cm propagation along the fiber. Right: Spectral evolution of the soliton, with according initial compensation of SSFS. See [4] and [15] for details.
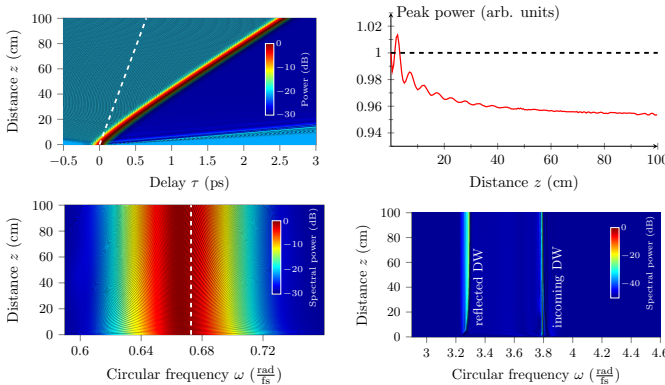

Fig. 4. Compensation of SSFS by a DW, approaching from the left. Top left: space-time trajectories. Top right: Evolution of soliton peak power, which is almost unchanged. Bottom: Spectral evolution of the soliton (left), and of the DW (right), which are locked now, c.f. Fig. 1. See [4], [15] for details.

On top of that, we demonstrate, how the interaction of solitons with DW's under conditions close to optical event horizons can be used to completely compensate the SSFS (Fig. 4). In this way, soliton trajectories can be completely stabilized [4]. Recently, there was an experimental verification that such effects really can take place, reported in [5]. In conclusion, a new method to manipulate optical pulses is presented. The method has been proven both numerically and experimentally.

REFERENCES

[1] S. Amiranashvili, U. Bandelow, and A. Mielke, "Calculation of ultrashort pulse propagation based on rational approximations for medium dispersion," *Opt. Quantum Electron.*, vol. 44, no. 3, pp. 241–246, 2012.

[2] S. Pickartz, U. Bandelow, and S. Amiranashvili, "Adiabatic theory of solitons fed by dispersive waves," *Phys. Rev. A*, vol. 94, p. 033811, 2016.

[3] ——, "Efficient all-optical control of solitons," *Opt Quant Electron*, vol. 48, pp. 503:1–7, 2016.

[4] ——, "Asymptotically stable compensation of soliton self-frequency shift," *Opt. Lett.*, vol. 42, no. 7, pp. 1416–1419, 2017.

[5] M. A. Eftekhar, Z. Sanjabi-Eznaveh, J. E. Antonio-Lopez, H. E. L. Aviles, S. Benis, M. Kolesik, A. Schülzgen, F. W. Wise, R. A. Correa, and D. N. Christodoulides, "Accelerating nonlinear interactions in tapered multimode fibers," in *Proc. CLEO 2018*, no. FTh1M.3. OSA, 2018.

[6] N. Akhmediev and A. Ankiewicz, *Solitons*, ser. Optical and Quantum Electronics. Chapman & Hall, 1997, vol. 5.

[7] U. Bandelow, A. Ankiewicz, S. Amiranashvili, and N. Akhmediev, "Sasa-Satsuma hierarchy of integrable evolution equations," *Chaos*, vol. in print, no. WIAS Preprint 2487, 2018.

[8] S. Amiranashvili, U. Bandelow, and N. Akhmediev, "Spectral properties of limiting solitons in optical fibers," *Optics Express*, vol. 22, no. 24, pp. 30 251–30 256, 2014.

[9] A. Sakovich and S. Sakovich, "The short pulse equation is integrable," *Journal of the Physical Society of Japan*, vol. 74, no. 1, pp. 239–241, Sep. 2005.

[10] S. Amiranashvili, M. Radziunas, U. Bandelow, and R. Ciegis, "Numerical methods for accurate description of ultrashort pulses in optical fibers," *Communications in Nonlinear Science and Numerical Simulation*, vol. 67, pp. 391–402, 2019.

[11] A. Demircan, S. Amiranashvili, and G. Steinmeyer, "Controlling light by light with an optical event horizon," *Phys. Rev. Lett.*, vol. 106, no. 16, p. 163901, April 2011.

[12] D. V. Skryabin and A. V. Yulin, "Theory of generation of new frequencies by mixing of solitons and dispersive waves in optical fibers," *Phys. Rev. E*, vol. 72, no. 1, p. 016619, 2005.

[13] A. V. Gorbach and D. V. Skryabin, "Light trapping in gravity-like potentials and expansion of supercontinuum spectra in photonic-crystal fibres," *Nature Photonics*, vol. 1, no. 11, pp. 653–657, 2007.

[14] T. G. Philbin, C. Kuklewicz, S. Robertson, S. Hill, F. Konig, and U. Leonhardt, "Fiber-optical analog of the event horizon," *Science*, vol. 319, no. 5868, pp. 1367–1370, 2008.

[15] S. Pickartz, "All-optical control of fiber solitons," Ph.D. dissertation, Humboldt University Berlin, Faculty of Science and Mathematics, 2018.

[16] J. P. Gordon, "Theory of the soliton self-frequency shift," *Optics Letters*, vol. 11, no. 10, pp. 662–664, Oct. 1986. [Online]. Available: https://www.osapublishing.org/viewmedia.cfm?uri=ol-11-10-662&seq=0&html=true