Numerical Simulation of Stokes Solitons in a Silica Microresonator

E. A. Anashkina [1],*, M. P. Marisova [1], G. Leuchs [1,2], A. V. Andrianov[1]
[1]Institute of Applied Physics of the Russian Academy of Sciences Nizhny Novgorod, 603950 Russia
[2]Max Planck Institute for the Science of Light, D-91058 Erlangen, Germany
* elena.anashkina@gmail.com

Abstract — **We report a novel generation regime of Stokes solitons numerically found in a silica microresonator in the framework of the generalized Raman-modified Lugiato-Lefever equation. These solitons can be attained for certain parameters in the anomalous dispersion range when the pump is in the normal dispersion range. We also demonstrate the Stokes soliton-like experimental spectrum similar to the numerically simulated spectrum.**

I. Introduction

Optical frequency combs (OFCs) generated in high *Q*-factor microresonators with whispering gallery modes are demanded for different applications and basic science [1]. OFCs based on Kerr nonlinearity can be obtained in manifold dynamical and steady-state regimes depending on system parameters [1,2]. Silica microspherical resonators with sufficiently high *Q*-factors can be produced in a cheap and easy way by melting the end of a telecom fiber [3,4], and the observed nonlinear effects in them can be described by the generalized Lugiato-Lefever equation as for other types of microresonators [1]. Such microresonators can be pumped by telecom C-band CW lasers through a tapered silica fiber, so the experimental scheme can be based on standard telecom components (Fig. 1(a)). However, despite the great attention paid to the study of intraresonator nonlinear dynamics, new effects are still being discovered [5].

Here we report a novel generation regime of Stokes (Raman-assisted) solitons numerically found in a silica microresonator. We also demonstrate the experimental spectrum similar to the numerically simulated spectrum. Note that numerical and experimental studies of the generation of Raman (Stokes) OFCs from a silica microresonator with controlled center frequency via detuning and optimization of coupling were reported in [6]. However, the possibility of Stokes soliton formation was not studied in [6]. A Stokes soliton co-existing with a dissipative Kerr soliton at the pump frequency in the anomalous dispersion range was discovered in [7]. But we could not find works reporting Stokes solitons in microresonators (in the anomalous dispersion range) when the pump with a weak OFC nearby was in the normal dispersion range.

II. Numerical model

To analyze the dynamics of the optical intraresonator field with allowance for the Raman nonlinearity, we used the generalized Raman-modified Lugiato-Lefever equation [1,5]:

$$t_R \frac{\partial E(t,\tau)}{\partial t} = \left[-\alpha - i\delta_0 + i\pi d \sum_{k\ge 2} \frac{\beta_k}{k!} \left(i\frac{\partial}{\partial \tau} \right)^k \right] E(t,\tau) \qquad (1)$$
$$+ i\gamma\pi d \cdot E(t,\tau) \int R(s)\, |E(t,\tau - s)|^2 \, ds + \sqrt{\theta} E_{in}$$

where $E(t, \tau)$ is the intraresonator field; t_R is the round trip time; t and τ are slow and fast times, respectively; α is the loss coefficient including intrinsic and coupling losses; β_k is the dispersion of the k-th order; γ is a nonlinear Kerr coefficient; d is a microsphere diameter; θ is the coupling coefficient; δ_0 is the frequency detuning of the pump field E_{in} from the exact resonance; and $R(t)$ is the Raman response function [8]:

$$R(t) = (1 - f_R)\delta(t) + f_R (T_1^{-2} + T_2^{-2}) T_1 \exp(-t/T_2) \sin(t/T) \qquad (2)$$

here $\delta(t)$ is the Dirac delta function; $f_R = 0.18$ is the fraction of Raman contribution to the total nonlinear response; $T_1 = 12.2$ fs, and $T_2 = 32$ fs. Further we use the normalized detuning and the normalized pump power X ($X = |E_{in}|^2 \cdot (\pi \cdot d \cdot \gamma \cdot \theta / \alpha^3)$ and $\Delta = \delta_0/\alpha$).

We used a home-made software based on the symmetrized split-step Fourier method (SSFM) [8] for modeling of intraresonator light field dynamics.

To find a free spectral range (FSR) and calculate dispersion of a microsphere (very important here) we solved numerically (by the modified Powell method) the characteristic equation for a TE mode family [9]

$$n \frac{\left[(kr)^{1/2} J_{l+1/2}(kr) \right]'}{(kr)^{1/2} J_{l+1/2}(kr)} = \frac{\left[(k_0 r)^{1/2} H^{(1)}_{l+1/2}(k_0 r) \right]'}{(k_0 r)^{1/2} H^{(1)}_{l+1/2}(k_0 r)} \qquad (3)$$

where the prime is the derivative with respect to the argument in parenthesis; r is a microsphere radius; $J_{l+1/2}$ and $H_{l+1/2}^{(1)}$ are the Bessel function and the Hankel function of the 1st kind of order l+1/2, respectively; l is the azimuthal index; $k_0 = 2 \cdot \pi \cdot \nu / c$ is the light propagation constant in vacuum; ν is the frequency; $k = n(\nu) \cdot k_0$; $n(\nu)$ is the linear refractive index of the silica glass (calculated using the Sellmeier formula given in [8]). We selected the first roots of Eq. (1) for the fundamental mode family. The iterative algorithm was implemented with the dispersion of the silica glass taken into account. After finding eigenfrequencies for the fundamental TE mode family, we calculate the 2nd-order dispersion

$$\beta_2 = -\frac{1}{2\pi^2 d} \frac{\Delta(\Delta\nu_l)}{(\Delta\nu_l)^3} \qquad (4)$$

where

$$\Delta\nu_l = \frac{\nu_{l+1} - \nu_{l-1}}{2}; \quad \Delta(\Delta\nu_l) = \nu_{l+1} - 2\nu_l + \nu_{l-1}. \tag{5}$$

We considered a silica microsphere with $d = 2r = 165$ μm. Its calculated dispersion is plotted in Fig. 1(b). The nonlinear coefficient $\gamma = 6.2$ (W·km)$^{-1}$ for $\nu \approx 193$ THz was calculated as in [10,11]. We neglected its frequency dependence in the simulations below.

III. RESULTS

The numerical investigation was motivated by the experimentally attained spectrum with a Stokes soliton-like shape located in the anomalous dispersion range in a 165-μm silica microsphere pumped in the normal dispersion range. The experimental scheme was similar to ones reported in [5,12], but certain parameters (mainly a coupling coefficient and a pump power) was specially adjusted.

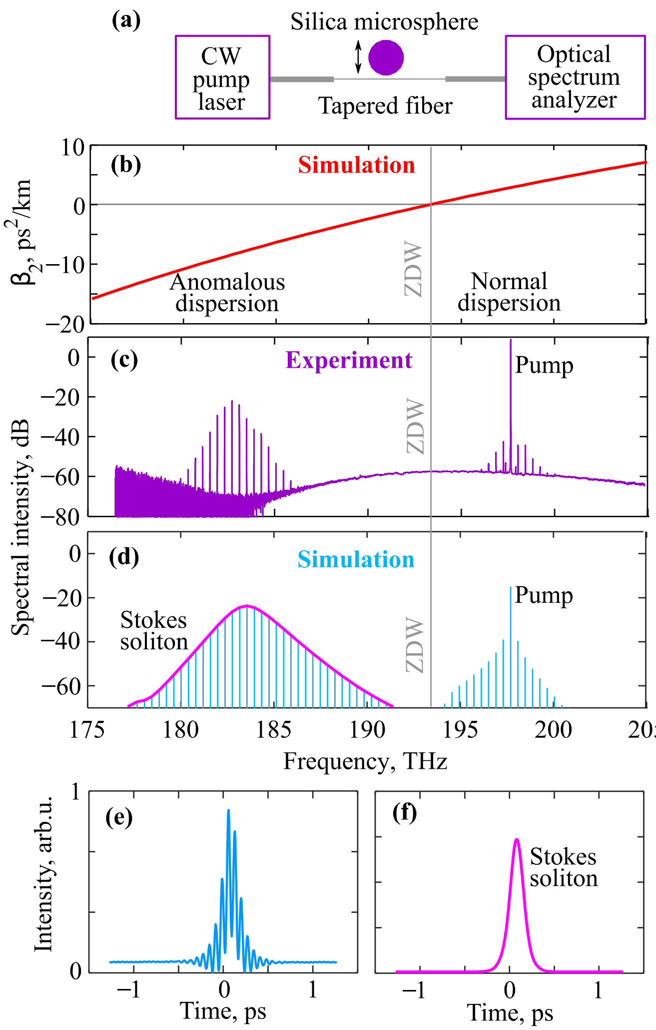

Fig. 1. (a). A simplified experimental scheme. Numerically calculated dispersion for a silica microsphere with a diameter of 165 μm. (c) The experimental spectrum demonstrating Stokes soliton-like OFC and (d) the corresponding numerically simulated spectrum. (e) Numerically simulated intensity distribution in the time domain. (f) Numerically simulated intensity distribution of the filtered out Stokes soliton in the time domain.

In the framework of the generalized Lugiato-Lefever equation with allowance for the Raman nonlinearity we found Stokes solitons in the anomalous dispersion range when the pump with a weak OFC nearby was in the normal dispersion range. We numerically modeled spectrum for $\Delta = 6$ and $X = 25$. A good agreement between the experimental (Fig. 1(c)) and the simulated (Fig 1. (d)) spectra was attained. The simulated intensity distribution in the time domain is shown in Fig. 1(d). Next, we filtered out the Stokes (Raman-assisted) OFC in the spectral domain and found its field distribution in the time domain. We indeed obtained the soliton demonstrated in Fig. 1 (e). So, we demonstrated the numerically the Stokes solitons in the anomalous dispersion range in a silica microsphere pumped in the normal dispersion range with a weak OFC near the pump frequency, for the first time, to the best of our knowledge.

ACKNOWLEDGMENT

The development of the testbed for experimental study was supported by the Mega-grant of the Ministry of Science and Higher Education of the Russian Federation, Contract No. 075-15-2021-633. The numerical and experimental studies of Stokes solitons were supported by the Russian Science Foundation, Grant No. 20-72-10188.

REFERENCES

[1]. A. Pasquazi *et al.*, "Micro-combs: A novel generation of optical sources," *Phys. Rep.*, vol. 729, pp. 1-81, 2018.

[2]. A. Coillet, Z. Qi, I. V. Balakireva, G. Lin, C. R. Menyuk, and Y. K. Chembo, "On the transition to secondary Kerr combs in whispering-gallery mode resonators," *Opt. Lett.*, vol. 44, pp. 3078-3081, 2019.

[3]. S. Spolitis *et al.*, "IM/DD WDM-PON communication system based on optical frequency comb generated in silica whispering gallery mode resonator," *IEEE Access*, vol. 9, pp. 66335-66345, 2021.

[4]. T. Salgals *et al.*, "Demonstration of a fiber optical communication system employing a silica microsphere-based OFC source," *Opt. Express*, vol. 29, pp. 10903-10913, 2021.

[5]. E. A. Anashkina, A. V. Andrianov, "Kerr-Raman optical frequency combs in silica microsphere pumped near zero dispersion wavelength," *IEEE Access*, vol. 9, pp. 6729-6734 (2021).

[6]. R. Suzuki, A. Kubota, A. Hori, S. Fujii, T. Tanabe, T. "Broadband gain induced Raman comb formation in a silica microresonator," *J. Opt. Soc. Am. B*, vol., pp. 933-938, 2018.

[7]. Q.-F Yang, X. Yi, K. Y. Yang, K. Vahala, K., "Stokes solitons in optical microcavities," *Nature Physics*, vol. 13, pp. 53–57, 2017.

[8]. G. P. Agrawal, *Nonlinear Fiber Optics*, 6th ed. Amsterdam, The Netherlands: Elsevier, 2019.

[9]. A. N. Oraevsky, "Whispering-gallery waves," *Quantum Electronics*, vol, 32, pp. 377-400, 2002.

[10]. E. A. Anashkina *et al.*, "Microsphere-based optical frequency comb generator for 200 GHz spaced WDM data transmission system," *Photonics*, vol. 7, 2020, Art. no. 72.

[11]. E. A. Anashkina, M. P. Marisova, A. A. Sorokin, A. V. Andrianov, "Numerical simulation of mid-infrared optical frequency comb generation in chalcogenide As_2S_3 microbubble resonators," *Photonics* vol. 6, 2019, Art. no. 55.

[12]. E. A. Anashkina *et al.*, "Kerr Optical Frequency Combs With Multi-FSR Mode Spacing in Silica Microspheres," *IEEE Photonics Technology Letters*, vol. 33, pp. 453-456, 2021.