Probability Theory of Single-Carrier Avalanche in HgCdTe APDs as a Stochastic Process

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***Abstract*—Recent researches have proven that HgCdTe is a good material to acquire both high multiplication and low excess noise factor at the same time in avalanche photodiodes (APDs). As a pseudo-binary narrow bandgap semiconductor material, HgCdTe exhibits high conduction band nonparabolicity as well as strong alloy scattering, especially for hot electrons, which changes the dynamics of hot electrons in a fundamental manner. Here, we propose a different scheme to characterize the scattering event and establish the probability theory, spatial description theory, to discuss the dynamics of electrons in HgCdTe APDs with the large nonparabolicity and the strong alloy scattering included. The spatial description theory is then compared with current analytic theory and the Monte Carlo method.**

Keywords—Spatial Description Theory, HgCdTe Avalanche Photodiodes, Analytic Methods, Excess Noise Factor

I. INTRODUCTION

In areas that require weak signal detection, especially in remote sensing , astronomy , quantum communication , and LIDARs , avalanche photodiodes (APDs) have been widely applied. [1] To improve the performance of APDs, especially to achieve higher gain and lower excess noise factors, a general idea is to find more suitable single-carrier avalanche material for respective applications. [2] In MWIR and LWIR, HgCdTe has already been one of the most important materials for its tunable bandgap from negative bandgap to near infrared [3], matured fabrication technology [4], and high absorption coefficient [5]. Researches have shown that both hot light hole and hot heavy hole exhibit very large effective mass [6], whereas the electron in the conduction band has a very small effective mass [7], which makes HgCdTe an ideal single-carrier avalanche material. As a pseudo-binary alloy, HgCdTe exhibits large alloy scattering, which is an isotropic scattering mechanism relaxing all the momentum within one scattering [8, 9], which is different from the polar optical phonon scattering mechanism that relaxes only a small amount of momentum [10], which is characterized by the classical hot carrier kinetical equation [9, 11]

$$\frac{dp}{dt} = eE - \frac{p}{\tau_m}, \qquad (1)$$

$$\frac{d\varepsilon}{dt} = eE\frac{p}{m} - \frac{\varepsilon - \varepsilon_0}{\tau_\varepsilon}, \qquad (2)$$

where τ_m , τ_ε are the momentum and energy relaxation times of the carrier, respectively. p and ε are the momentum and energy of the carrier respectively. ε_0 is the average energy of the cold electrons. m and e are the mass and charge carried by the carrier. E is the external electric field. For the case of alloy scattering, this equation is only satisfied for a group of carriers by averaging the momentum of these carriers. [10] If the positions of carriers are unknown, the operation of averaging is degraded into a spatial integral, which is denoted as the temporal description paradigm below. For a homogeneous system, this integral could be performed without loss of information. In the multiplication region in HgCdTe APDs, the electric field, impurity concentration, and perhaps the Cd component are changing with the spatial coordinate, which will pull the system very far from a homogeneous system and needs additional attention. [12]

The excess noise factor of the single carrier avalanche photodiode, such as HgCdTe APDs, should tend to a constant value when the gain is large enough. [13, 14] However, the mid-infrared HgCdTe APDs exhibit an additional rise of excess noise factor for a large gain. [12, 15] Based on the spatial description theory, a generalized form of the history-dependent model of avalanche by R. J. McIntyre is introduced. [12] With this generalized form, the gain and excess noise factor of the light and dark current could be derived separately and the value of the total excess noise factor is derived only by considering the law of large number and the rise of the value is found when the gain is high enough.

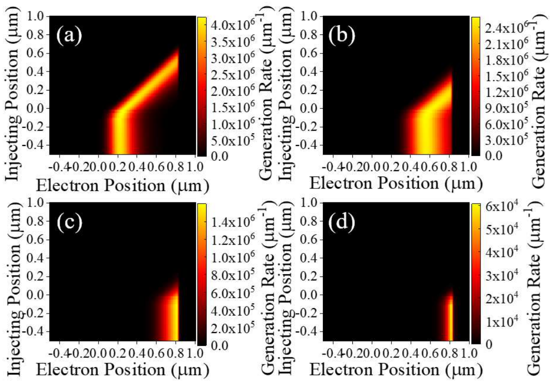

Fig. 1. Impact ionization at different position. (a) the 1st impact ionization. [16] (b) the 2nd impact ionization. [16] (c) the 3rd impact ionization. [16] (d) the 4th impact ionization.

II. The Necessity of Spatial Description Theory

As is shown in Sec. I, the high scattering rate of and the fast relaxation of momentum by the alloy scattering bring additional constraints to the dynamical equation of hot electrons. Within the spatial description theory, the trajectory of any carrier is spitted into two types of sub-trajectories, the forward motion, and the backward motion. The forward motion includes the small-angle scattering, such as the polar optical phonon scattering, and the effective small angle, such as the part of trajectory after an obstacle angle scattering when the angle between the momentum and the electric field is turned to acute angle again by the external electric field. The backward motion includes the trajectory after the obstacle angle scattering and between two forward motion trajectories.

Besides the reason above, there are still two aspects that this spatial description theory should be introduced: Firstly, the result of manipulation of doping, concentrations are accurate to the atomic scale, whereas the sampling rate in most measurement is too low to observe the microscopic process. Thus, rather than a temporal description, the spatial description is the theory to analyze semiconductors in accordance with the nature of measurement. Secondly, with the spatial description theory, different microscopic processes could be considered together compactly and accurately without truncating higher orders, which infers that the spatial description theory is neat and elegant in modeling the avalanche phenomenon.

A comparison between current analytic theories, Monte Carlo methods, and spatial theory presented in this work is shown in Tab. I.

TABLE I
COMPARING WITH OTHER AVAILABLE METHODS

Theoretical Aspects			
	Current Analytic	Monte Carlo	This Work
Paradigm	Temporal	None	Spatial
Describe avalanche	Deterministic or Probability Theory	Statistics	Probability Theory
Backward-scattering	Qualitative	Considered in some works	Quantitively discussion. Corrections are presented
Nonparabolicity	Effective	As a scattering correction	Combined into equations of motion
Energy transfer in impact ionization	Phenomenological	Inaccurate in some works	The concept of the ghost area is present. Correction is derived
Application Aspects			
	Current Analytic	Monte Carlo	This Work
Fitting Parameters	Yes	Yes or No	No
Calculation Speed	Fast	Slow	Fast
Distribution of Impact Ionization	No	Statistical Estimated	Accurate Probability Density
Dark Current Analysis	Inaccurate due to dynamics of hot electron inaccurate	Inaccurate due to the partial avalanche negelected.at present	Accurate

III. Conclusion

In this work, a probability theory of single-carrier avalanche photodiode, spatial description theory, is presented with a detailed discussion of the necessity of the theory. Albeit the discussion presented in this work is mainly aimed at the mid-infrared HgCdTe APDs, the spatial description theory, as well as the generalized form of the history-dependent model of avalanche, may have a wider area of application in APDs of other materials and transport of hot electrons in narrow bandgap semiconductor.

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