Moving Bragg Solitons in a Coupler with Separated Grating and Cubic-Quintic Nonlinearity

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***Abstract*—We investigate the existence and stability of moving solitons a semilinear directional coupler where one core has cubic-quintic nonlinearity and the other core is linear with uniform Bragg grating.**

***Index Terms*—Moving Bragg solitons; Bragg grating; Cubic-quintic nonlinearity;**

I. Introduction

Fiber Bragg gratings (FBGs) are widely known to exhibit strong effective dispersion in a medium, up to 10^6 times higher than the chromatic dispersion of silica fiber [1]. When this strong dispersion is counterbalanced by the third-order nonlinearity, soliton-like structures form in the system [2–4] which are termed as "Bragg solitons." Theoretically, Bragg solitons may possess any velocity from zero to the speed of light in the medium. Experimentally, Bragg solitons with speeds as low as 16% of the speed of light in vacuum have been observed [5]. The analysis of the characteristics of Bragg solitons has received much interest in recent years due to their potential applications in signal processing, switching, buffering and logic operations [6–8].

Coupling between modes in nonlinear optical systems such as nonlinear couplers with dissimilar cores gives rise to rich nonlinear dynamics and switching characteristics [9–11]. In this work, we analyze the existence and stability of moving Bragg solitons in a semilinear coupler where one core has qubic-quintic nonlinearity and the other core is linear with a uniform FBG.

II. The Model

The propagation of light in a coupler made of a nonlinear core with cubic-quintic nonlinearity and a linear core with a uniform Bragg grating is described by the the following set of normalized equations

$$\begin{aligned} &iu_t + iu_x + \left[|v|^2 + \frac{1}{2}|u|^2\right] u - q \left[\frac{1}{4}|u|^4 + \right. \\ &\qquad \left. \frac{3}{2}|u|^2|v|^2 + \frac{3}{4}|v|^4\right] u + \phi = 0. \\ &iv_t - iv_x + \left[|u|^2 + \frac{1}{2}|v|^2\right] v - q \left[\frac{1}{4}|v|^4 + \right. \\ &\qquad \left. \frac{3}{2}|v|^2|u|^2 + \frac{3}{4}|u|^4\right] v + \psi = 0, \\ &\qquad i\phi_t + ic\phi_x + u + \lambda\psi = 0, \\ &\qquad i\psi_t - ic\psi_x + v + \lambda\phi = 0. \end{aligned} \tag{1}$$

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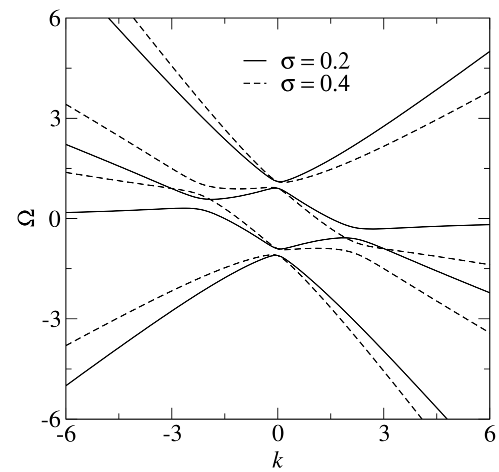

Fig. 1. Linear spectra for different velocities at $\lambda = 0.2$ and $c = 0.2$.

Here, u and v are the forward- and backward-propagating waves in the nonlinear core (core-1) while ϕ and ψ are their counterparts in the linear core with FBG (core-2). $q > 0$ represents the strength of the quintic nonlinearity and $\lambda > 0$ is the coupling coefficient between forward and backward-waves in the linear core. The coefficient of mutual coupling between the two cores has been normalized to 1. c denotes the group velocity mismatch between the cores and the group velocity in core-1 has been set to 1.

To determine the spectrum within which solitons may exist, Eqs. (1) are first transformed to the moving coordinates using the transformation $\{X, T\} = \{x - \sigma t, t\}$, where σ represents the normalized soliton velocity. This leads to the following system of equations:

$$\begin{aligned} &iu_T + i(1-\sigma)u_X + \left[|v|^2 + \frac{1}{2}|u|^2\right] u - \\ &q\left[\frac{1}{4}|u|^4 + \frac{3}{2}|u|^2|v|^2 + \frac{3}{4}|v|^4\right] u + \phi = 0, \\ &iv_T - i(1+\sigma)v_X + \left[|u|^2 + \frac{1}{2}|v|^2\right] v - \\ &q\left[\frac{1}{4}|v|^4 + \frac{3}{2}|v|^2|u|^2 + \frac{3}{4}|u|^4\right] v + \psi = 0, \\ &i\phi_T + i(c-\sigma)\phi_X + u + \lambda\psi = 0, \\ &i\psi_T - i(c+\sigma)\psi_X + v + \lambda\phi = 0. \end{aligned} \tag{2}$$

The linear spectrum of the system is obtained by substituting $u, v, \phi, \psi \sim e^{i(kX - \Omega T)}$ into the linearized form of Eqs. (2),

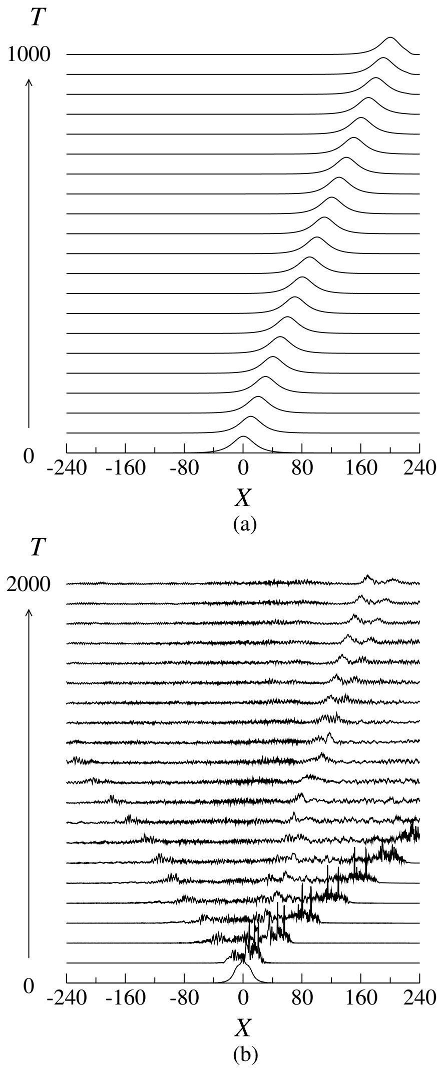

Fig. 2. Propagation of solitons with $\lambda = 0.2$, $c = 0.2$ and $\sigma = 0.2$ showing: (a) stable Type 1 soliton for $\Omega = 1.09$, $q = 0.26$; (b) unstable Type 2 soliton for $\Omega = -1.05$, $q = 0.7$.

where Ω is the frequency of the moving frame and k is the wavenumber. As is shown in Fig. 1, the spectrum generally consists of three disjoint bandgaps of which only the upper and lower bandgaps contain soliton solutions. Additionally, in the upper and lower bandgaps, two disjoint families of solitons are found, namely Type 1 and Type 2, which differ in amplitude and phase.

As for moving soliton solutions, Eqs. (2) do not have analytical solutions and they must be solved by numerical methods.

III. Propagation and Stability Analysis

Examples of the propagation of stable and unstable solitons are shown in Fig. 2 and a summary of the stability analysis is presented in Fig. 3, for fixed values λ, c and σ, on the (q, Ω) plane. Stable solitons are only found in the Type 1 family, in both the upper and lower bandgaps while all Type 2 solitons are found to be unstable. Also, there exist regions

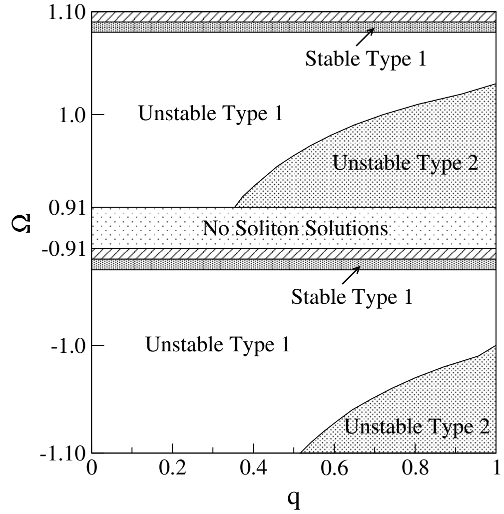

Fig. 3. Stability diagram for moving soliton corresponding to $\sigma = 0.2$, $\lambda = 0.2$ and $c = 0.2$ on the (q, Ω) plane.

in the upper and lower bandgaps (see diagonal lines in Fig. 3) where solitons do not exist.

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