

THEORY AND MODELLING OF HIGH-FIELD CARRIER TRANSPORT IN HIGH-SPEED PHOTODETECTORS

Nick Zakhleniuk

Photonics Research Group CES Department, University of Essex, Colchester, UK

(Inter-University EPSRC UK Project - PORTRAIT)

• **New Microscopic and Macroscopic Theory of High-Field Transport in Inhomogeneous Electric Fields (Including Built-in Fields)**

•**Implications for Physics-Based Software Modelling Tools**

•**Application to p-i-n Photodiodes: Steady State and Transient**

•**Transient Simulation of UTC Photodiodes: New Underlying Physics, Consequences, and Interpretation of the Results**

•**Conclusions**

High-Field Transport in Photodetectors

Department of **Computing and** Electronic Systems \rightarrow

University of Essex

Speed of response – transient time:

$$
\tau_{tr} = \frac{w}{v}, \quad v = \mu F.
$$

Transient Time and Electron Dynamics in Built-in Electric Fields

Department of **Computing and Electronic Systems**

TRANSIT TIME AND HIGH-FIELD MOBILITY

$$
\tau_{tr} = \frac{w}{v}, \qquad v = \mu F, \qquad \mu = \frac{e \tau_p}{m^*}.
$$

•**In high electric fields:**

 $\mu \,=\, \mu\,(\,F\,$).

•**Usually** μ **decreases when F increases, and drift velocity becomes sub-linear function of F.** •**Saturation velocity and transferred electron model:**

$$
\mu(F) = \frac{\mu_{low}}{\left[1 + \left(\frac{\mu_{low}F}{v_{sat}}\right)^{\beta}\right]^{1/\beta}}.
$$

$$
\mu(F) = \frac{\mu_{low} + \left(\frac{v_{sat}}{F}\right)\left(\frac{F}{F_0}\right)^4}{1 + \left(\frac{F}{F_0}\right)^4}.
$$

•**Drift velocity and mobility of holes << then for e-ns.**

Velocity-Field Characteristics for e-ns and holes in Bulk (Homogeneous Fields)

T. Ishibashi et al, (Invited Review Paper) "High-Speed and High-Output InP-InGaAs Unitravelling-Carrier Photodiodes", IEEE J. Select. Topics Quantum Electron, Vol. 10, 709, (2004).

I University of Essex Theory of Carrier Transport in Photodetectors

Department of **Computing and Electronic Systems**

THE SIMPLEST (BUT NOT THE BEST!) MODEL IS THE DRIFT-DIFFUSION (DD) MODEL:

$$
\frac{\partial n}{\partial t} - \frac{1}{e} \nabla_x \mathbf{j}_n(x) = \alpha \frac{I_0}{\hbar \omega_0} e^{-\alpha x} - R(n, p),
$$

$$
\frac{\partial p}{\partial t} + \frac{1}{e} \nabla_x \mathbf{j}_h(x) = \alpha \frac{P_0}{\hbar \omega_0} e^{-\alpha x} - R(n, p),
$$

•**The goal is to calculate e/h current densities j n and j h for a given optical intensity I 0 (W/cm 2), i.e. to calculate the electrical response of photo-**, p), **diode, and all local parameters.**

$$
\nabla_{x}E_{c}(x) = \frac{e^{2}}{\epsilon\epsilon_{0}}[p(x) - n(x) + N_{D}^{+}(x) - N_{A}^{-}(x)],
$$
\n
$$
j_{n}(x) = n(x)\mu_{n}(x)\nabla_{x}E_{c}(x) + eD_{n}(x)\nabla_{x}n(x) = n(x)\mu_{n}(x)\nabla_{x}E_{Fn}(x),
$$
\n
$$
j_{h}(x) = p(x)\mu_{h}(x)\nabla_{x}E_{c}(x) - eD_{h}(x)\nabla_{x}p(x) = p(x)\mu_{h}(x)\nabla_{x}E_{Fn}(x),
$$
\n
$$
n(x) = N_{c}(T_{0})F_{1/2}(\eta_{n}(x)), \quad \eta_{n}(x) = [E_{Fn}(x) - E_{c}(x)]/k_{0}T_{0},
$$
\n
$$
p(x) = N_{v}(T_{0})F_{1/2}(\eta_{h}(x)), \quad \eta_{h}(x) = [E_{v}(x) - E_{Fn}(x)]/k_{0}T_{0}.
$$

•**The feature of p-i-n and UTC PDs is the presence of strong built-in field.** •**The key question is: What does define the local mobility** ^μ**n,h(x)?**

Electron Dynamics in Built-in Electric Fields – University of Essex **Key Previous Results**

Department of **Computing and Electronic Systems**

JOURNAL OF APPLIED PHYSICS

VOLUME 39, NUMBER 10

SEPTEMBER 1968

Transport of Electrons in a Strong Built-in Electric Field

I. B. GUNN

IBM Watson Research Center, Yorktown Heights, New York 10598 (Received 4 March 1968; in final form 13 May 1968)

The problem is discussed of the transport of electrons in the presence of a strong field due to fixed space charges, as in the depletion region of a $p-n$ junction. It is found that the effective mobility, for small departures from equilibrium, is equal to the chordal hot-electron mobility which would result if the strong field were applied by external means.

$\mu = \mu(F_{\textit{Buil}-\textit{in}}) .$ The result:

JOURNAL OF APPLIED PHYSICS

VOLUME 40, NUMBER 11

OCTOBER 1969

Carrier Heating or Cooling in a Strong Built-in Electric Field

R. STRATTON

Texas Instruments Incorporated, Dallas, Texas 75222 (Received 6 March 1969; in final form 4 June 1969)

A recently published calculation for the effective carrier mobility in a semiconductor space charge region with a strong built-in field is shown to lead to an unphysical result. A method for deriving the volt-current characteristic for small perturbations from equilibrium is presented.

 μ = μ $_{low}$. The result:

•**The above results are obtained for systems near the equilibrium.** •**The results are in disagreement with each other. THE QUESTIONS:** •**What is a correct result? What is a mobility in systems with built-in fields far from equilibrium, which mobility e.g. should be used in DD theory?**

In Case 2 $\Phi_{\scriptscriptstyle{0}}(\vec{p},\vec{r})$ is unknown, and finding it presents the most difficult part. $\Phi_{\circ}(\vec{p}, \vec{r})$

| University of Esex | General Results for Mobility at High and Low | Computation | |
|--|--|--|---|
| 1. High Carrier Density: | \n $\mu(T_n) \Rightarrow \mu[R(f)], T_n \Rightarrow R(f), f(\vec{r}) = \vec{V}, E_c(\vec{r}), \vec{V}, E_F(\vec{r}),$ \n | | |
| 1. High Carrier Density: | \n $\mu(T_n) \Rightarrow \mu[R(f)], T_n \Rightarrow R(f), f(\vec{r}) = \vec{V}, E_c(\vec{r}), \vec{V}, E_F(\vec{r}),$ \n | | |
| where $T_n = R(f)$ is given by the solution of <u>simplified</u> balance equation: | | | |
| \n $\mu(T_n) \vec{\nabla}_r E_c(\vec{r}), \vec{\nabla}_r E_F(\vec{r}) = eW(T_n, T_o)$ \n | \n $W(T_n, T_o)$ \n | is the power loss function. | |
| 2. Low Carrier Density: | \n Inhomogeneous field solution: | | |
| \n $\Phi_0(\vec{p}, F_0)$ \n | \n Substitution\n $F_c \rightarrow [f(\vec{r})/e]^{1/2}$ \n | \n $\Phi_0(\vec{p}, F_0) \Rightarrow \Phi_0[\vec{p}, f(\vec{r})]$ \n | \n $f(\vec{r}) = \vec{v}, E_c(\vec{r}), \vec{v}, E_F(\vec{r})$ \n |
| \n $\mu(F_o) \Rightarrow \mu(f/e)^{1/2}$ \n | \n $\mu(F_o) \Rightarrow \mu(f/e)^{1/2}$ \n | | |

| Results of the Mobility Calculations for | Department of Computing and Computing and Electronic Systems | |
|---|--|---|
| Case of High Electron Density: | For a particular model with: | $\mu(T_n) = \mu_0 \times (T_0/T_n)$ and $W(T_n, T_0) = (3/2)k_0(T_n - T_0)/\tau_c$ the high-field mobility is: |
| $\mu(f) = \mu_0 / [1/2 + \sqrt{1/4 + \alpha_0 f}], \quad f(\vec{r}) > 0;$ \n <td>$\mu(f) = \mu_0, \quad f(\vec{r}) < 0.$\n</td> \n <td>$\alpha_0 = 2\mu_0 \tau_c / 3ek_0 T_0.$\n</td> \n | $\mu(f) = \mu_0, \quad f(\vec{r}) < 0.$ \n | $\alpha_0 = 2\mu_0 \tau_c / 3ek_0 T_0.$ \n |
| Case of Low Electron Density (The saturation velocity model): | | |
| $\mu(F) = \frac{\mu_0}{\mu_0} \quad \text{if } \mu_0$ \n | $\mu(f) = \frac{\mu_0}{\mu_0} \quad \text{if } \mu_0$ \n | $\mu(f) = \frac{\mu_0}{\mu_0} \quad \text{if } \mu_0$ \n |

$$
\mu(F_o) = \frac{\mu_o}{[1 + (\mu_o F_o / v_{sat})^{\beta}]^{1/\beta}} \quad \Rightarrow \quad \mu(f) = \frac{\mu_o}{[1 + (\mu_o f^{1/2}(\vec{r})/e^{1/2}v_{sat})^{\beta}]^{1/\beta}}, \quad \text{if} \quad f(\vec{r}) > 0;
$$
\n
$$
\mu(f) = \mu_o \equiv \mu_{low}, \quad \text{if} \quad f(\vec{r}) < 0.
$$

cooling (decrease of T_n). In this case approximately $\mu(f) \approx \mu_0.$

La University of Essex Calculated Mobilities Using Various Models

Department of **Computing and Electronic Systems**

•Large difference in the high-field mobilities for DD models using various driving forces clearly shows that μ(F) and μ(gradE_{Fermi}) are not equivalent.

• μ(f) from the DD theory qualitatively follows $\mu(T_n)$ from the HD theory. This means that DD is also able to correctly describe the HF transport, provided that the correct DF is used.

•Curves 1 and 2 obtained from DD simulation using $\mu(F) = \mu_o / \sqrt{1 + (\mu_o F / v_{sat})^2}$

with (1) $F = |\nabla_x E_F(x)/e|$ and (2) $F = |\nabla_x E_c(x)/e|$.

- Curves 1' and 2' are calculated mobilities using $F = [f(x)/e^2]^{1/2}$, where $f(x) = \nabla_x E_F(x) \cdot \nabla_x E_c(x)$ was obtained from the results corresponding to curves 1 and 2, respectively.
- •Curves HD1 and HD2 are the results of full HD simulation using $\mu(T_n) = \mu_o(T_o / T_n)$ (HD1) and $\mu(T_n) = \mu_0 /[\sqrt{1 + \kappa^2 (T_n - T_o)^2 + \kappa (T_n - T_o)}]$ (HD2), where $\kappa = 3 \mu_0 k_o / 4 e \tau_o v_s^2$. (HD2), where $\kappa = 3\mu_o k_o / 4e\tau_g v_s^2$

Hydrodynamic or Energy Balance Model University of Essex

Department of **Computing and Electronic Systems**

•**Additional energy flux continuity (balance) equations for e-ns and holes:**

$$
\frac{\partial w_n}{\partial t} + \vec{\nabla} \cdot \vec{S}_n = \frac{1}{e} \vec{J}_n \cdot \vec{\nabla} E_c + \frac{w_n - w_0}{\tau_{\varepsilon}},
$$

\n
$$
\vec{S}_n = -\frac{3}{2} \frac{k_0 T_n}{e} (\vec{J}_n + k_0 n \mu_n \vec{\nabla} T_n),
$$

\n
$$
w_n = \frac{3}{2} n k_0 T_n,
$$

\n
$$
\mu_n = \mu_0 \times (T_0 / T_n),
$$

\n
$$
\mu_n = \mu_0 / \left[\sqrt{1 + \kappa_n^2 (T_n - T_0)^2} + \kappa_n (T_n - T_0) \right] \kappa_n = \frac{3}{4} \frac{k_0 \mu_0}{e \tau_{\varepsilon} v_s^2}.
$$

•**Important difference: DD model does not includes hot-electron effects, the HD model does.**

•**Boundary conditions at all interfaces are formulated via TE emission carrier fluxes and the carrier energy fluxes.**

Calculated Electric Fields And Field Parameter

•In general the field parameter f(r) does not follow F(r) or $\nabla \mathsf{E}_\mathsf{F}(\mathsf{r}).$

Department of

Computing and Electronic Systems

•Profiles of the electric field $|\nabla_x E_c(x)/e|$ (dash-dotted lines);

•Profiles of the "field parameter" $[f(x)/e]^{1/2}$ (solid lines);

•Profile of the gradient of QFL $|\nabla_x E_F(x)/e|$ (dashed line, only curve 1 is shown).

Application to Simulation of p-i-n Photodiode: University of Essex **Transient Analysis: Mobility**

Department of **Computing and Electronic Systems**

Transient mobility profiles at t=1.5 ps for various models: DD1 – F, DD2 - VE_F, HD – T_n, TC – Theoretical Calculations.

In spite of large difference in the drift mobility/velocity profiles (previous figure), the output signals are very close for all models. What is a physical reason for this result?

Application to Simulation of p-i-n Photodiode: University of Essex Transient Analysis: Density and Temperature

Department of **Computing and Electronic Systems**

Excess electron density at various t. Electron temperature at various t.

At high optical excitations the current is determined by the TE emission at the IF rather than by the high-field transport. The electron diffusion current flows away from the n-InP/InGaAs IF and it helps to counterbalance the fast drift supply of the electrons to the IF. Although the max of T n(x) is shifted away from the IF as *t* increases, at *t~2 ps* T $_{\sf n}$ (x=0.5)=360 K is still higher than T $_{\sf o}$ and this explains faster response in HD model.

Department of **Computing and
Electronic Systems** \rightarrow

THEORY AND SIMULATIONS OF UTCPHOTODETECTORS

High-Speed Response of Uni-Traveling-Carrier Photodiodes Tadao ISHIBASHI, Satoshi KODAMA, Naofumi SHIMIZU and Tomofumi FURUTA NTT System Electronics Laboratories, Morinosato-Wakamiya 3-1, Atsugi, Kanagawa 243-01, Japan

Jpn. J. Appl. Phys. Vol. 36 (1997) pp. 6263-6268 Part 1, No. 10, October 1997

Built-in Electric Field

Department of Computing and
Electronic Systems

Department of **I** University of Essex Strong Electric Field and Strong Gradient Computing and

•**When equilibrium is violated, the highfield carrier dynamics is determined by the joint action of F (** ∇ **E c) and** ∇**n, NOT by the electric field alone. This is the physical reason behind introduction of the field parameter f(r):**

$$
f(\vec{r}) = \vec{\nabla}_r E_c(\vec{r}).\vec{\nabla}_r E_F(\vec{r}),
$$

Department of **Computing and Electronic Systems**

T. Ishibashi et al, (Invited Review Paper) "High-Speed and High-Output InP-InGaAs Unitravelling-Carrier Photodiodes", IEEE J. Select. Topics Quantum Electron, Vol. 10, 709, (2004).

Response of UTC PD (A=2500 μ **m2) to 1.55** μ**m incident pulse with FWHM=0.4 ps. (2pJ input energy corresponds to intensity ≈ I_o=1x10⁵ W/cm 2). The experiment is well described with the HD model, but not with the DD model.**

University of Essex

(For comparison: Photoresponse of a p-i-n PD. The experimental curves are well described by the simulation results with the DD or the HD models).

۴ **Effect of Bias on Photoesponse: HD, I=1x10 5 W/cm 2**

Evolution of Electron Temperature at V ⁿ=0 V

Evolution of Electron Temperature at V ⁿ=1 V

Thermionic Emission Current from the Absorber

Department of Computing and
Electronic Systems

DESSIS UG:

$$
J_{n,2}=J_{n,1}
$$

$$
J_{n,2} = aq \bigg[v_{n,2} n_2 - \frac{m_2}{m_1} v_{n,1} n_1 \exp \left(-\frac{\Delta E_C}{k_B T_{e,1}} \right) \bigg]
$$

$$
S_{n,2} = S_{n,1} + \frac{c}{q} J_{n,2} \Delta E_C
$$

$$
S_{n,2} = (-b) \bigg[v_{n,2} n_2 k_B T_{e,2} - \frac{m_2}{m_1} v_{n,1} n_1 k_B T_{e,1} \exp \left(-\frac{\Delta E_C}{k_B T_{e,1}} \right) \bigg]
$$

where the 'emission velocities' are defined as:

$$
v_{n,i} = \sqrt{\frac{k_B T_{e,i}}{2\pi m_i}}
$$

Transidiers Indian Concilled Cooling of Carriers in Absorber

ED University of Essex Electron Density and Electron Mobility at the Cooling

Example 3 University of Essex Effect of the Mobility in Absorber on Photoresponse

Department of **Computing and
Electronic Systems**

Example 2 University of Essex Effect of the Optical Intensity on the Electron Mobility

Department of **Example 2 University of Essex Effect of the Optical Intensity on the Electron Density Computing and** Electronic Systems \rightarrow $t = 1.25$ ps $\sqrt{t=3.0}$ ps 10^{17} 10^{17} $V_n = 0 V$ $V_n = 0 V$ 10^{16} 5 10^{16} 5° Electron Density (cm³) Electron Density (cm³) $\overline{\mathbf{4}}$ 10^{15} $\overline{\mathbf{4}}$ 10^{15} $\overline{3}$ 10^{14} $\mathbf{3}$ 10^{14} Opt Intensity: **Optical Intensity:** $\overline{2}$ 1 - 10^2 W/cm² $1 - 10^2$ W/cm² 10^{13} $\overline{2}$ 10^{13} $2 - 10^3$ W/cm² $2 - 10^3$ W/cm² $3 - 10⁴$ W/cm² $3 - 10^4$ W/cm² 1 10^{12} $4 - 10^5$ W/cm² 10^{12} $4 - 10^{5}$ W/cm²

 1.1

 $\mathbf{1}$

 $5 - 5x10^5$ W/cm²

 0.7

 0.9

 $Y(\mu m)$

 0.8

 0.6

 10^{11} \overline{u}

 $5 - 5x10^5$ W/cm²

 0.7

 0.9

 $\mathbf{1}$

 1.1

 $\mathbf{0.8}$

 Y (µm)

 10^{11}

 0.6

University of Essex Physics of Transient Carrier Cooling in Absorber

•**Process 1 – Excess of the excitation energy goes in part to the lattice (via the phonon emission) and in part into the electron system (via the e-e scattering). These processes are very fast (~10 fs).**

•**Process 2 – There is no energy excess and the energy is taken from the lattice in order to heat the electron up to the mean energy k 0T ⁿ. This process is slower than the first one and it is governed by the energy relaxation time** $\tau_{\scriptscriptstyle{\text{g}}}\text{)}.$

•**The cooling below T n (or even below T 0) is possible only for a Process 2.**

Department of **Computing and Electronic Systems**

University of Essex Mathematics of Transient Carrier Cooling in Absorber

 $\frac{1}{\mathbf{c}}\vec{\mathbf{J}}_{\text{n}}\cdot\vec{\nabla}\text{E}_{\text{c}} + \frac{\text{w}_{\text{n}}-\text{w}_{\text{0}}}{\tau}.$

Department of Computing and **Electronic Systems**

$$
\frac{\partial n}{\partial t} - \frac{1}{e} \nabla_x \mathbf{j}_n(x) = \alpha \frac{I_0}{\hbar \omega_0} e^{-\alpha x} - R(n, p),
$$

$$
\frac{\partial p}{\partial t} + \frac{1}{e} \nabla_x \mathbf{j}_h(x) = \alpha \frac{P_0}{\hbar \omega_0} e^{-\alpha x} - R(n, p).
$$

 W_{n} $\vec{\nabla}$ \vec{C} $\vec{\nabla}$ \vec{I} $\vec{\nabla}$ $\vec{\nabla}$ \vec{C} \vec{C}

 \rightarrow \rightarrow \rightarrow \rightarrow

 $+\vec{\nabla}\cdot\vec{S}_n = -\vec{J}_n \cdot \vec{\nabla} E_n + \frac{w_n - \vec{S}_n}{\sqrt{n}}$

n n c

 $\frac{1}{\hbar} + \vec{\nabla} \cdot \vec{S}_n = \frac{1}{e}$

n

∂

∂

•**Where is the term** [∝] ^α**I0 in the energy balance equation, which describes the optical energy supply?**

•**In order to obtain this term the Boltzmann equation must be modified in the first instance, since the above equations were obtained from the BE (as the moments):**

$$
\left[\vec{\nabla}_{\mathbf{x}}\mathbf{E}_{\mathbf{c}}(\mathbf{x})\cdot\vec{\nabla}_{\mathbf{p}_{\mathbf{x}}} + \vec{\mathbf{v}}_{\mathbf{p}_{\mathbf{x}}}\cdot\vec{\nabla}_{\mathbf{x}}\right]\Phi(\vec{\mathbf{p}},\mathbf{x}) = \mathbf{\hat{I}}\Phi(\vec{\mathbf{p}},\mathbf{x}) + \alpha \frac{\mathbf{I}_{0}}{\hbar\omega_{0}}e^{-\alpha\mathbf{x}}\frac{1}{g(E)}\delta[E - (\hbar\omega_{0} - E_{g})].
$$

•**Integration of the latter equation (…** [×]**E) results in the following EB equation:**

ετ

$$
\frac{\partial w_{n}}{\partial t} + \vec{\nabla} \cdot \vec{S}_{n} = \frac{1}{e} \vec{J}_{n} \cdot \vec{\nabla} E_{c} + \frac{w_{n} - w_{0}}{\tau_{\epsilon}} + \alpha \frac{I_{0}}{\hbar \omega_{0}} e^{-\alpha x} \Lambda_{eff} (\hbar \omega_{0} - E_{g}).
$$

•**The absence (erroneous!) of the last term from the energy balance equation means that the resulting equation describes a very particular case of photo excitation: ħ** ω **⁰=E g. This corresponds to Process 2 and is the real reason for the cooling.**

Comparison of p-i-n and UTC

Department of **Computing and Electronic Systems**

Temperature Profiles

Results from DD Modelling of the UTC PD

Department of Ę **Computing and Example 2 Independing on Photoresponse in the DD Model** Electronic Systems \rightarrow $1E-06$ 1E-06 .10 V $10V$ 9E-07 9E-07 **Optical Intensity: Optical Intensity:** 8E-07 8E-07 **I0=1x10 5 W/cm 2I0=1x10 5 W/cm 2** $5\overline{)}$ 5 Output Current (A/µm) 7E-07 $\overline{3}$ 3 6E-07 $\overline{2}$ 5E-07 4E-07 'o v $0V$ 3E-07 $2E-07$ 2E-07 $1E-07$ $1E-07$ 5E-12 $1E-11$ $1.5E-11$ $2E-11$ $2.5E-11$ $3E-11$ $\mathbf 0$ 2E-12 4E-12 6E-12 8E-12 $1E-11$ $\bf{0}$ Time (s) Time (s)

En University of Essex Effect of Different Driving Forces on Photoresponse

Output Currents from DD Model at Very High Photoexcitation Level for Various Biases

University of Essex

Effect of the Photoexcitation Pulse Duration on University of Essex **the Output Current for DD Model**

Department of **Computing and Electronic Systems**

•**In case of spatially-inhomogeneous electric fields, which are typical for most of semiconductor devices, the driving force for carrier drift mobility in the drift-diffusion model is the fieldp parameter:**

$$
f(\vec{r}) = \vec{\nabla}_r E_c(\vec{r}).\vec{\nabla}_r E_F(\vec{r}): \quad \mu = \mu(f).
$$

•The available mobility models, like v_{sat} or transferred electrons, must **be modified respectively, in order to include the field parameter.**

•**In the high-speed photodetectors the hot-electrons effects are of paramount importance for the fast transient responses.**

•**The HD model must be used for simulation of p-i-n or UTC photodetectors, since it includes the hot-electron effects.**

•**For devices with photoexcitation the optical sources (optical heating/cooling) must be included in the energy balance equations.**

•**The key role of the near-interface field-heating of the carriers in the absorber is shown in the fast response of the UTC PDs.**