

Successful Application of
the 8-band $k\cdot p$ Framework to
Optical Properties of Highly Strained
In(Ga)As/InGaAs Quantum Wells with
Strong Conduction-Valence Band Coupling

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1. Background – Highly strained quantum wells
2. $\mathbf{k} \cdot \mathbf{p}$ theory
3. Purpose
4. Analytical methods
5. Numerical and experimental results
6. Conclusion

1. Background – Highly strained quantum wells (QWs)

Infrared light source for measuring environmental gases

Single mode lasing, mW output power, and wavelength tunability

→ Bandgap tuning of QW laser for conventional optical fiber communication

→ Larger compressive strain in well layer

Semiconductor lasers with highly strained In(Ga)As QWs on InP substrates

	Well	Strain [%]	Lasing wavelength [μm]
1	InGaAs	1.65	2.07
2	InGaAs	1.85	2.103
3	InGaAs	1.94	2.13
4	InAs	3.2	2.33

1. M. Mitsuhashi *et al.*, APL, vol. 72, pp.3106, 1998.
2. T. Sato *et al.*, APL, vol. 87, 211903, 2005.
3. T. Sato *et al.*, JSTQE, vol. 13, pp.1079, 2007.
4. T. Sato *et al.*, EL, vol. 43, pp. 1143, 2007.

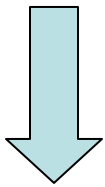
2. $k \cdot p$ theory

$k \cdot p$ theory – Band structure analysis of semiconductors

- Gain (Absorption), Scattering rate, Photoluminescence (PL) spectra

Total Hamiltonian of unstrained system

$$H_{total} = H_{k \cdot p} + H_{s.o.}$$



Transformation to strained system

- Coordinate transformation
- Expansion up to first order of strain

Total Hamiltonian of strained system

$$H_{total} = H_{k \cdot p} + H_{s.o.} + D_{k \cdot p} + D_{s.o.}$$
$$\approx H_{k \cdot p} + H_{s.o.} + D_{k \cdot p}$$

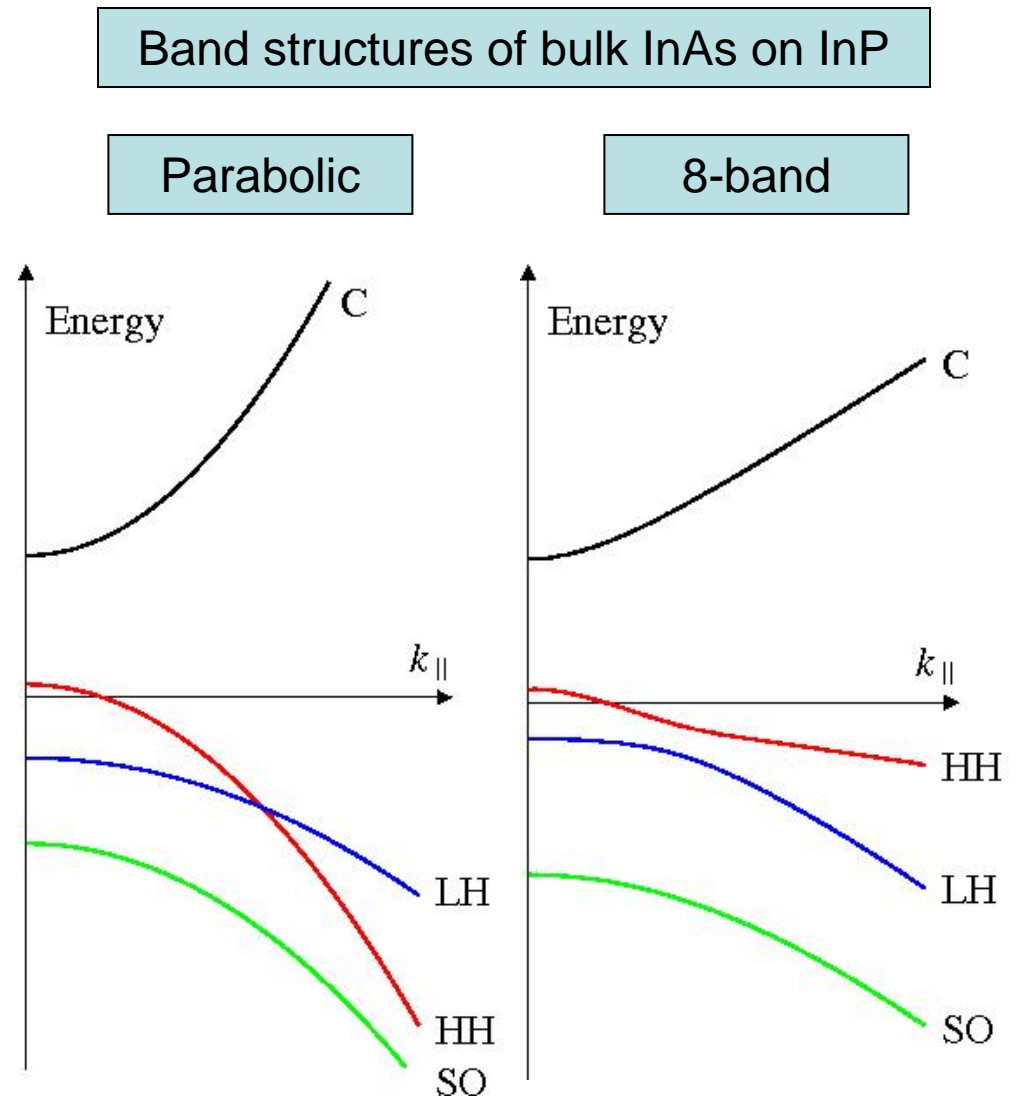
$D_{s.o.}$ is usually neglected in conventional $k \cdot p$ analysis

	Hamiltonian originating from
H_{total}	Total system
$H_{k \cdot p}$	$k \cdot p$ perturbation
$H_{s.o.}$	Spin-orbit coupling
$D_{k \cdot p}$	Strain and $H_{k \cdot p}$
$D_{s.o.}$	Strain and $H_{s.o.}$

2-1. Effective-mass approximation

Band	Parabolic	4-band
C	$m_{C,eff}$	$m_{C,eff}$
HH	$m_{HH,eff}$	Coupling
LH	$m_{LH,eff}$	
SO	$m_{SO,eff}$	$m_{SO,eff}$
Band	6-band	8-band
C	$m_{C,eff}$	Coupling
HH	Coupling	
LH		
SO		

C: Conduction band
 HH: Heavy-hole band
 LH: Light-hole band
 SO: Spin-orbit split-off band



2-2. 6- and 8-band model

Block-diagonalized Hamiltonian (QW, $k_t = 0$)

$H_{k \cdot p} + H_{s.o.}$ (Hamiltonian of unstrained system)

$$\begin{array}{cccc|l}
 \underbrace{E_g + \frac{\hbar^2 k_z^2}{2m_0}}_{\text{blue circle}} & 0 & \underbrace{-i\sqrt{\frac{2}{3}}P_0k_z}_{\text{red circle}} & \underbrace{-i\sqrt{\frac{1}{3}}P_0k_z}_{\text{red circle}} & |C\rangle \\
 0 & -\frac{\hbar^2}{2m_0}(\gamma_1 - 2\gamma_2)k_z^2 & 0 & 0 & |HH\rangle \\
 \underbrace{i\sqrt{\frac{2}{3}}P_0k_z}_{\text{red circle}} & 0 & \underbrace{-\frac{\hbar^2}{2m_0}(\gamma_1 + 2\gamma_2)k_z^2}_{\text{Valence band}} & -\frac{\hbar^2}{m_0}\gamma_2k_z^2 & |LH\rangle \\
 \underbrace{i\sqrt{\frac{1}{3}}P_0k_z}_{\text{red circle}} & 0 & -\frac{\hbar^2}{m_0}\gamma_2k_z^2 & -\frac{\hbar^2}{2m_0}\gamma_1k_z^2 - \Delta & |SO\rangle
 \end{array}$$

For 6-band model, terms surrounded by ○, are phenomenologically incorporated into the term surrounded by ○, as the effective mass of an electron.

2-2. 6- and 8-band model

Block-diagonalized Hamiltonian (QW, $k_t = 0$)

$D_{k \cdot p}$ (Hamiltonian of strained system)

$$\begin{bmatrix}
 a_c \varepsilon & 0 & -i \sqrt{\frac{2}{3}} P_0 \frac{2c_{12}}{c_{11}} k_z \varepsilon & -i \sqrt{\frac{1}{3}} P_0 \frac{2c_{12}}{c_{11}} k_z \varepsilon \\
 0 & -p - q & 0 & 0 \\
 i \sqrt{\frac{2}{3}} P_0 \frac{2c_{12}}{c_{11}} k_z \varepsilon & 0 & -p + q & \sqrt{2}q \\
 i \sqrt{\frac{1}{3}} P_0 \frac{2c_{12}}{c_{11}} k_z \varepsilon & 0 & \sqrt{2}q & -p
 \end{bmatrix}
 \begin{matrix}
 |C\rangle \\
 |HH\rangle \\
 |LH\rangle \\
 |SO\rangle
 \end{matrix}$$

Valence band

- Terms surrounded by \bigcirc , are **NOT** taken into account in 6-band model.
- Terms surrounded by \bigcirc , are linear in terms of strain, ε , and k_z .
- For QWs with larger strain, the difference between 6- and 8-band model is increased.
- **The strain at which the 6-band model becomes inappropriate is unclear.**

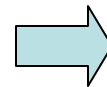
2-3. Interaction between strain and spin-orbit coupling

Block-diagonalized Hamiltonian (QW)

$D_{s.o.}$ (Hamiltonian of strained system)

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{2}{3}\Delta\varepsilon & 0 & 0 \\ 0 & 0 & -\frac{2}{9}\Delta\left(1 - \frac{4c_{12}}{c_{11}}\right)\varepsilon & -\frac{\sqrt{2}}{9}\Delta\left(1 + \frac{2c_{12}}{c_{11}}\right)\varepsilon \\ 0 & 0 & -\frac{\sqrt{2}}{9}\Delta\left(1 + \frac{2c_{12}}{c_{11}}\right)\varepsilon & \frac{8}{9}\Delta\left(1 - \frac{c_{12}}{c_{11}}\right)\varepsilon \end{bmatrix} \begin{matrix} |C\rangle \\ |HH\rangle \\ |LH\rangle \\ |SO\rangle \end{matrix}$$

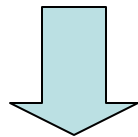
- Discarded in the conventional analysis
- Independent of in-plane wavenumber, k_t
- Constant energy shift of valence bands
- Larger energy shift for larger strain



Non-negligible energy shift
for highly strained quantum
wells

For QWs with larger strain

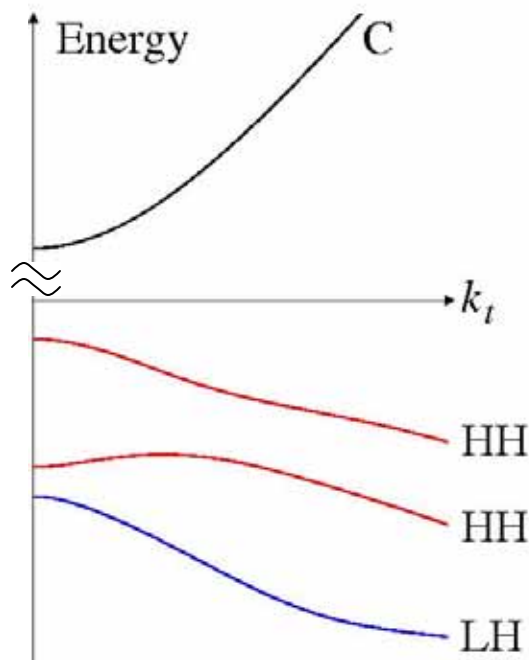
- 6- or 8-band model?
- Effect of interaction between spin-orbit coupling and strain ($D_{s.o.}$)



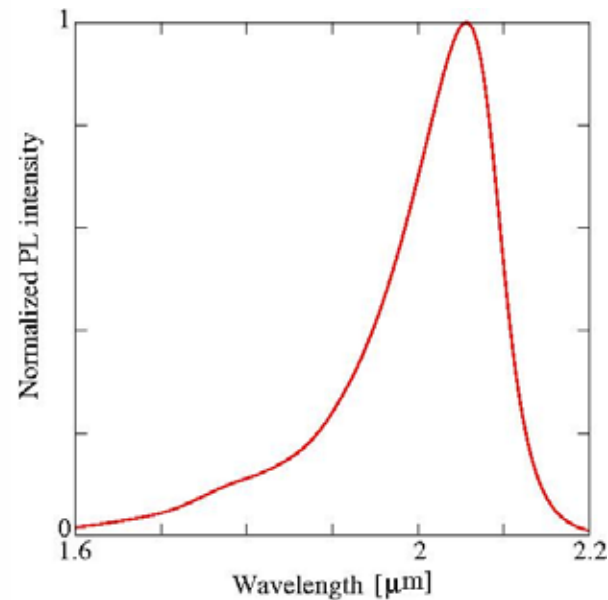
- Highly strained In(Ga)As/InGaAs QWs on InP
- Strain dependence of band edge optical properties
 - PL spectra
 - Absorption spectra

4. Analytical methods

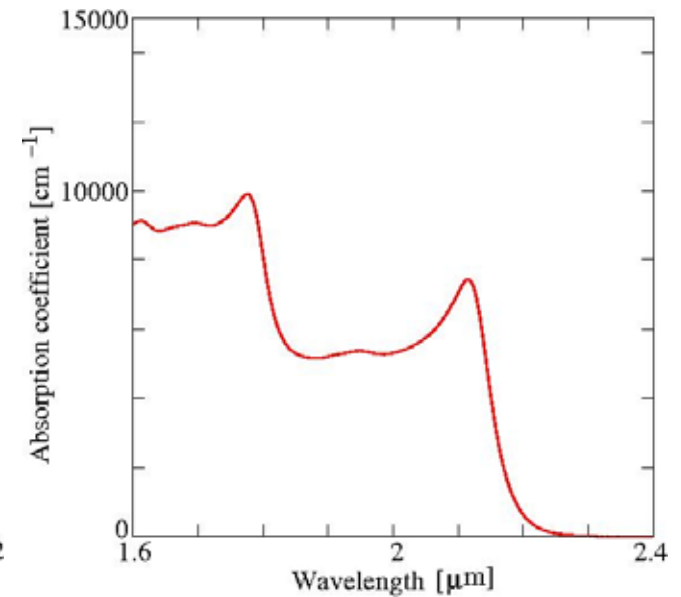
Band structure



PL spectra



Absorption



- Band structure – 8×8 $\mathbf{k} \cdot \mathbf{p}$ Hamiltonian
 - Finite-difference method
- PL spectra – Fermi's golden rule
 - sech lineshape broadening
- Absorption spectra – Non-variational approach (exciton effects)

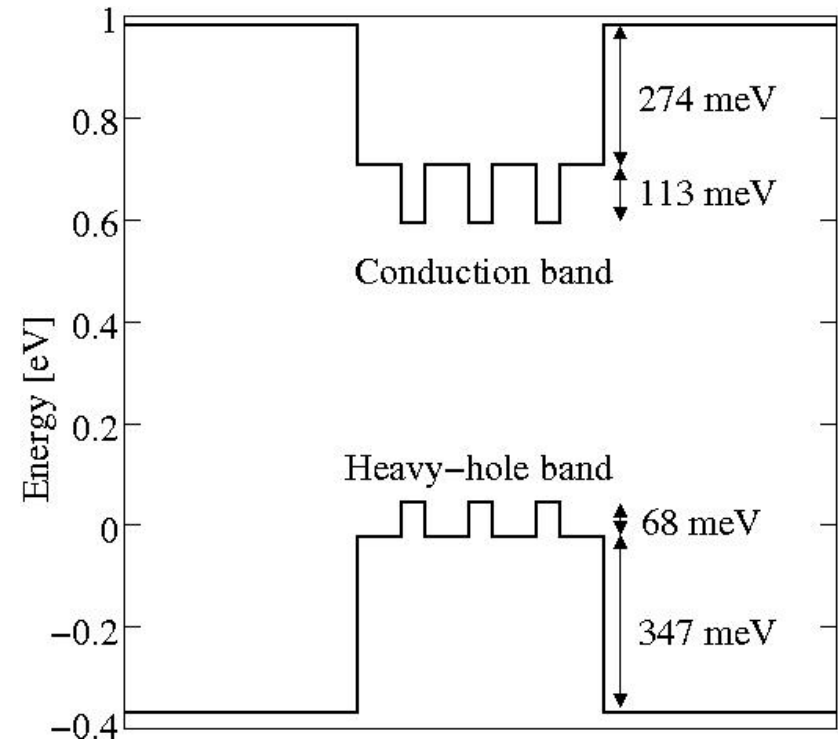
5-1. 6- or 8-band model?

Highly strained InGaAs/InGaAs QWs ($L_w = 10$ nm, $\epsilon = 1.65\%$)

Layer structure

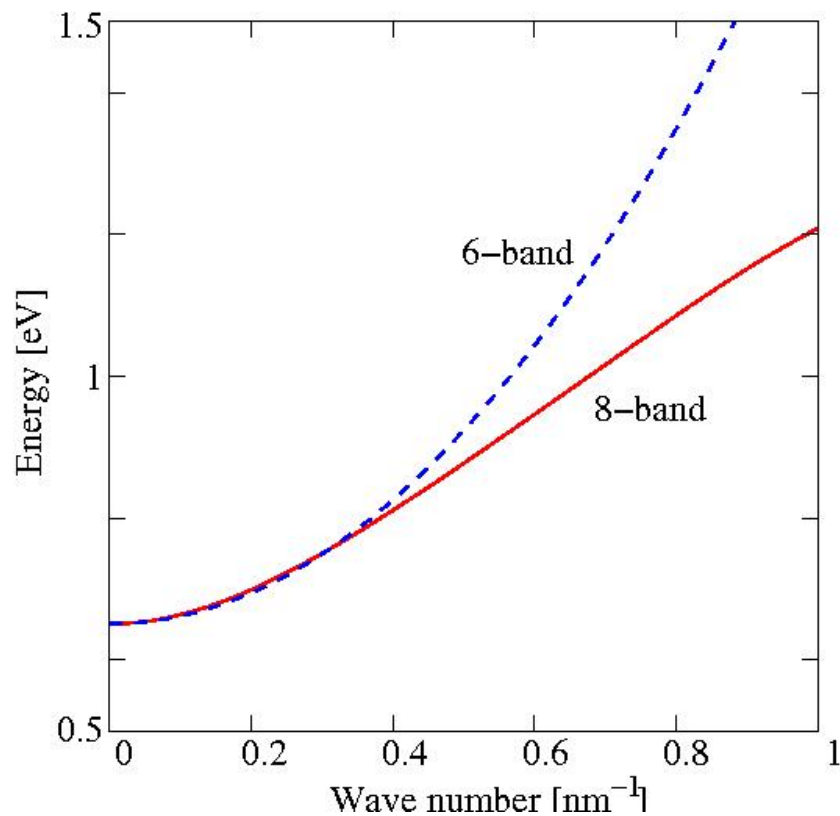
InP
InGaAs Barrier 0% 19 nm
InGaAs Well 1.65% 10 nm
InGaAs Barrier 0% 19 nm
InGaAs Well 1.65% 10 nm
InGaAs Barrier 0% 19 nm
InGaAs Well 1.65% 10 nm
InGaAs Barrier 0% 19 nm
InP

Potential profile

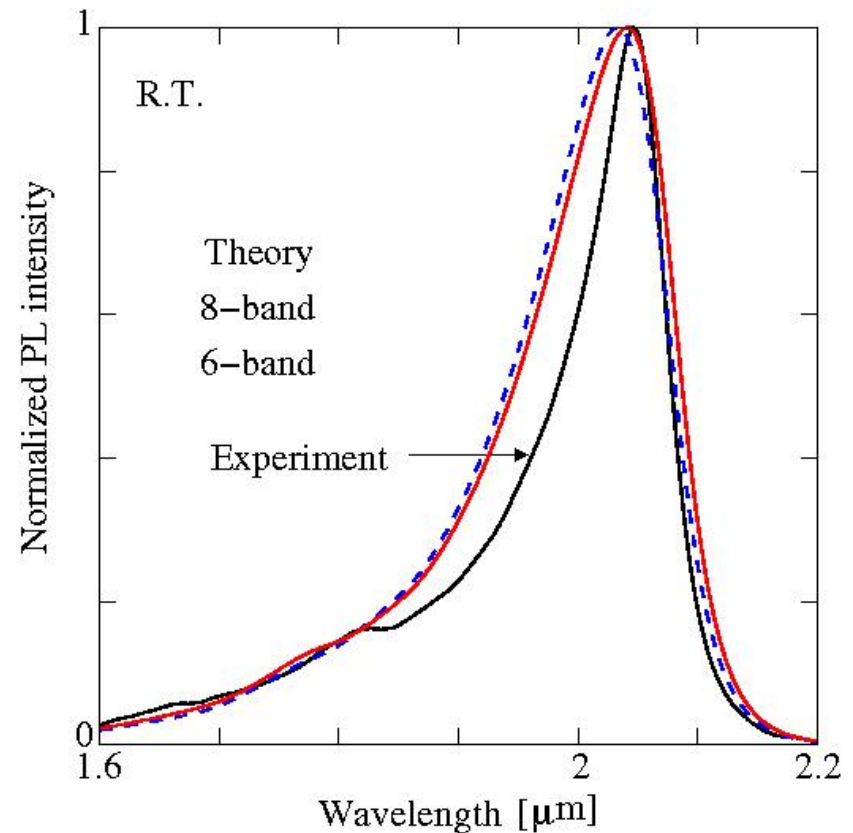


5-1. 6- or 8-band model?

Conduction band structure



PL spectra



- Around Γ point, almost the same conduction band structure for both models
- About PL spectra, good agreement with experiment for both models

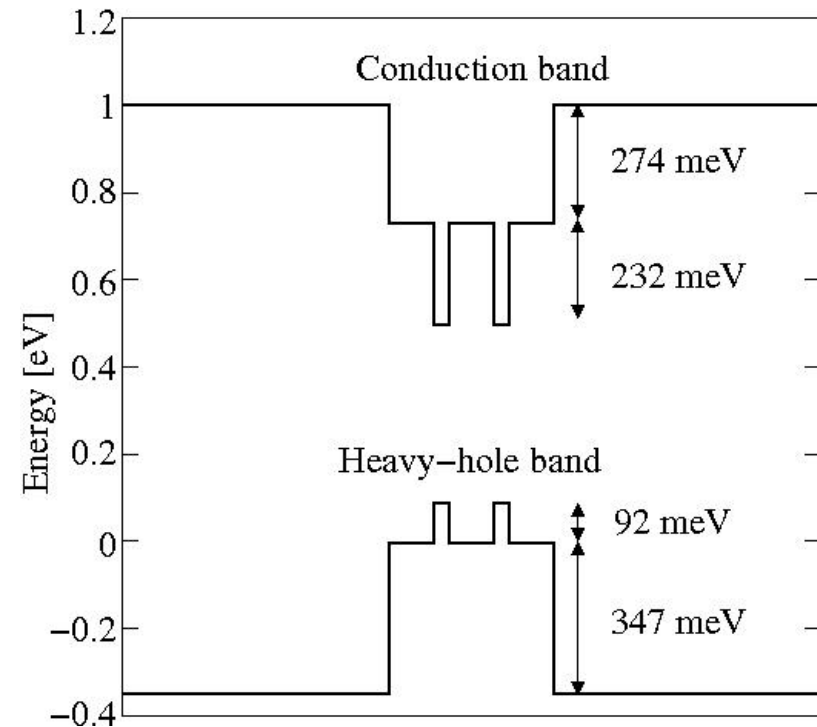
5-1. 6- or 8-band model?

Highly strained InAs/InGaAs QWs ($L_w = 6$ nm, $\varepsilon = 3.2\%$)

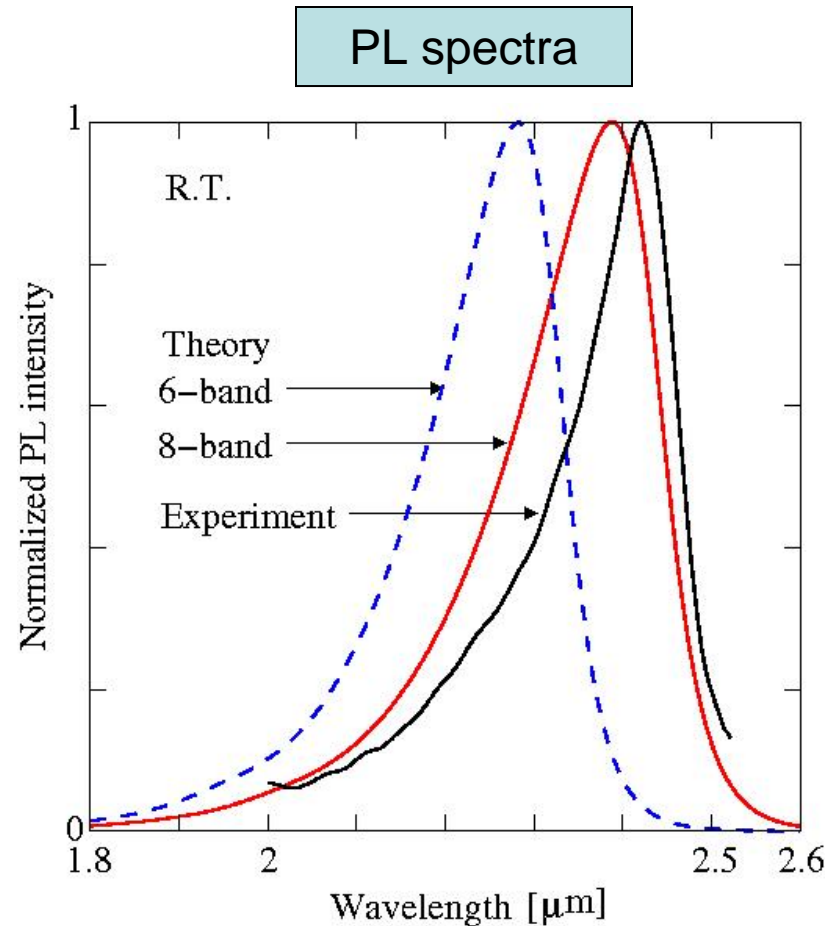
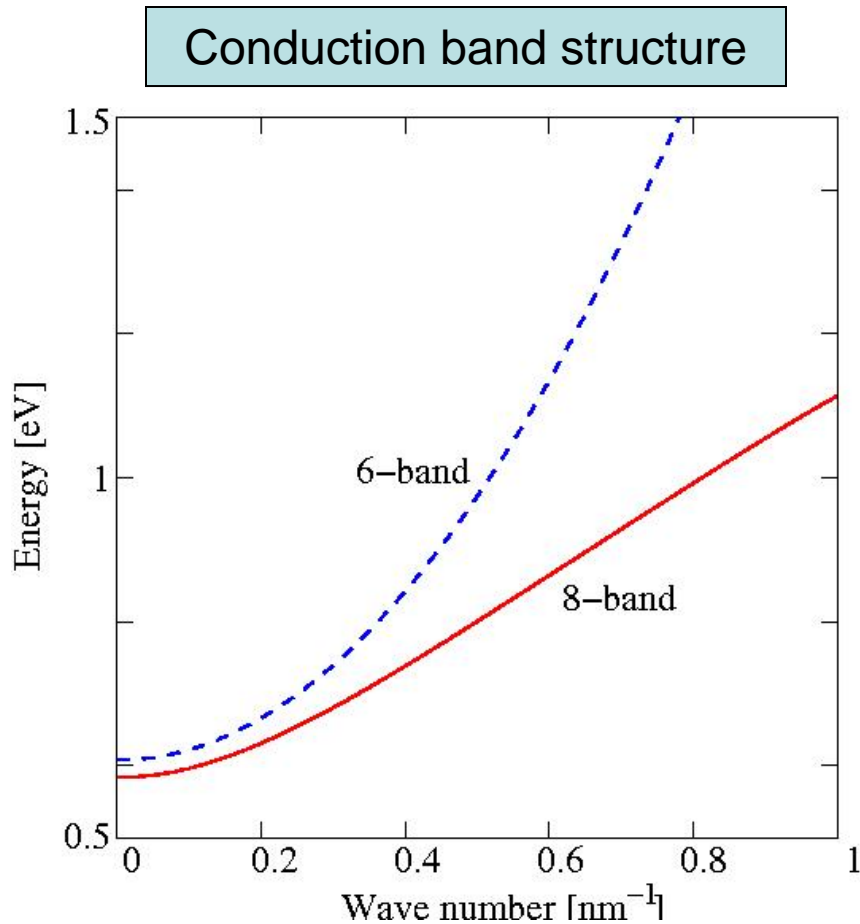
Layer structure

InP
InGaAs Barrier 0% 16.5 nm
InAs Well 3.2% 6 nm
InGaAs Barrier 0% 16.5 nm
InAs Well 3.2% 6 nm
InGaAs Barrier 0% 16.5 nm
InP

Potential profile

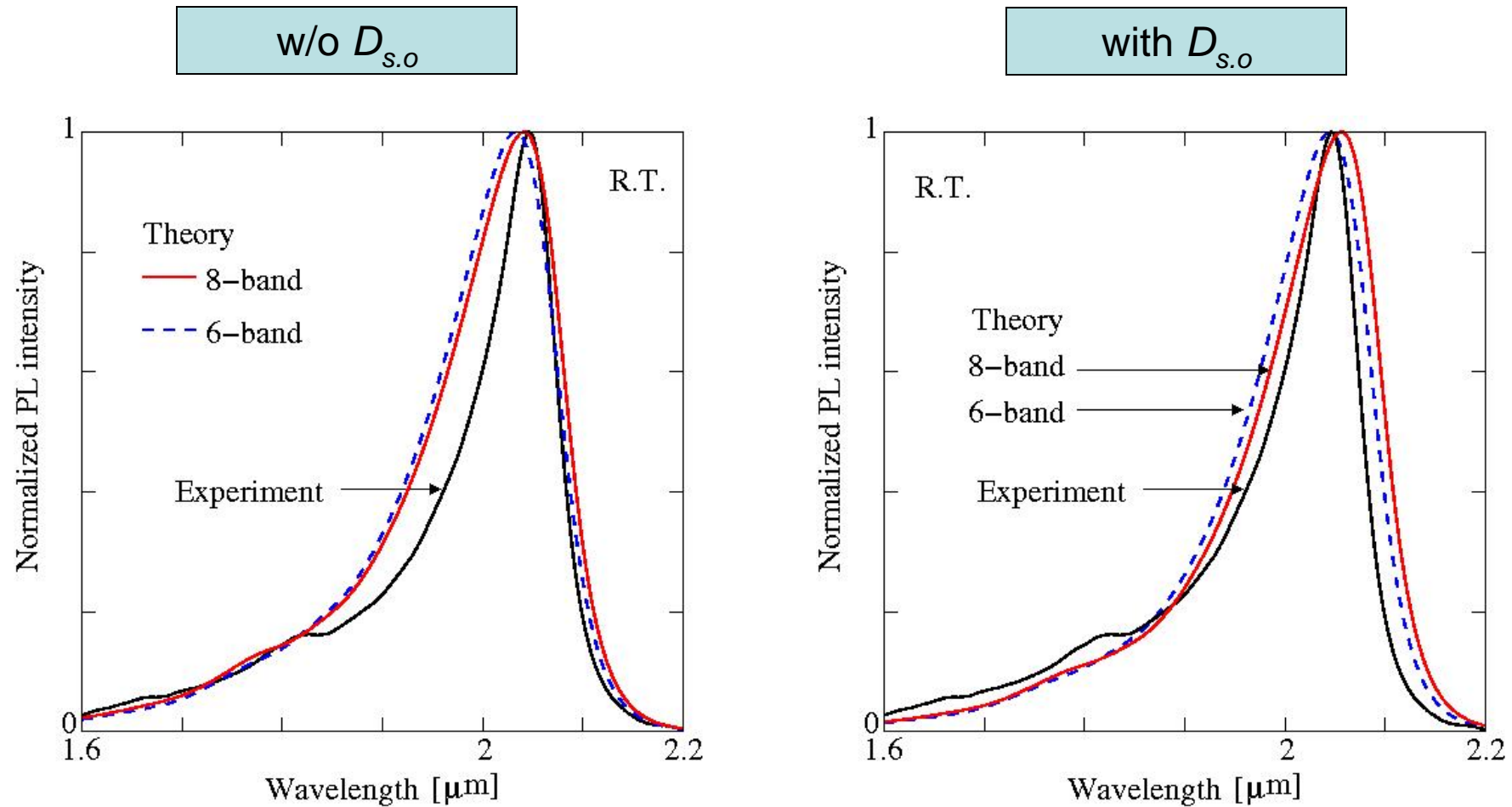


5-1. 6- or 8-band model?



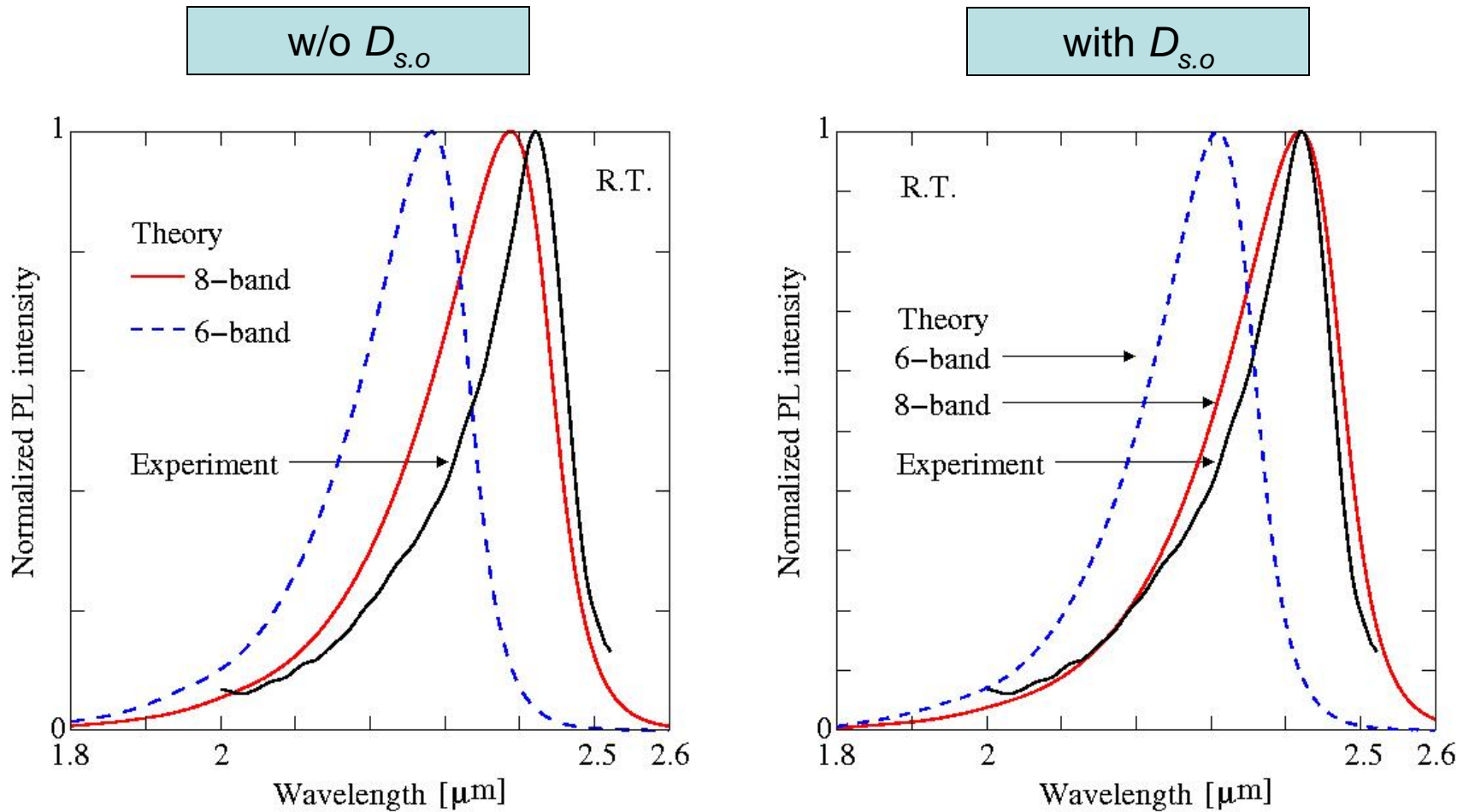
- Non-negligible energy shift around Γ point for 8-band model
- For 6-band model, PL peak wavelength is 150 nm too short
- About PL spectra, good agreement with experiment for 8-band model

5-2. Effect of $D_{s.o.}$



- InGaAs/InGaAs QWs ($L_w = 10$ nm, $\varepsilon = 1.65\%$)
- 10-nm red shift in calculated PL spectrum

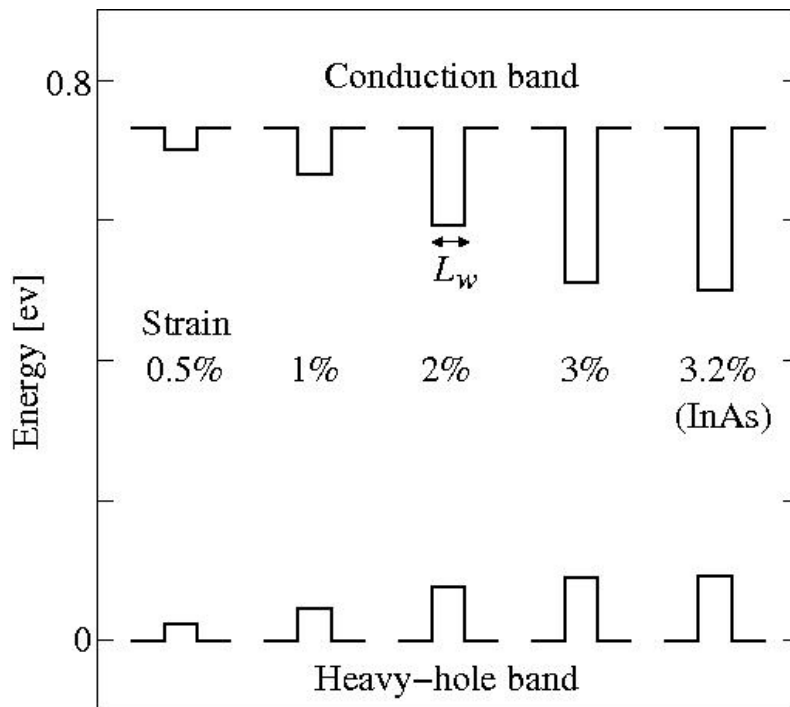
5-2. Effect of $D_{s.o.}$



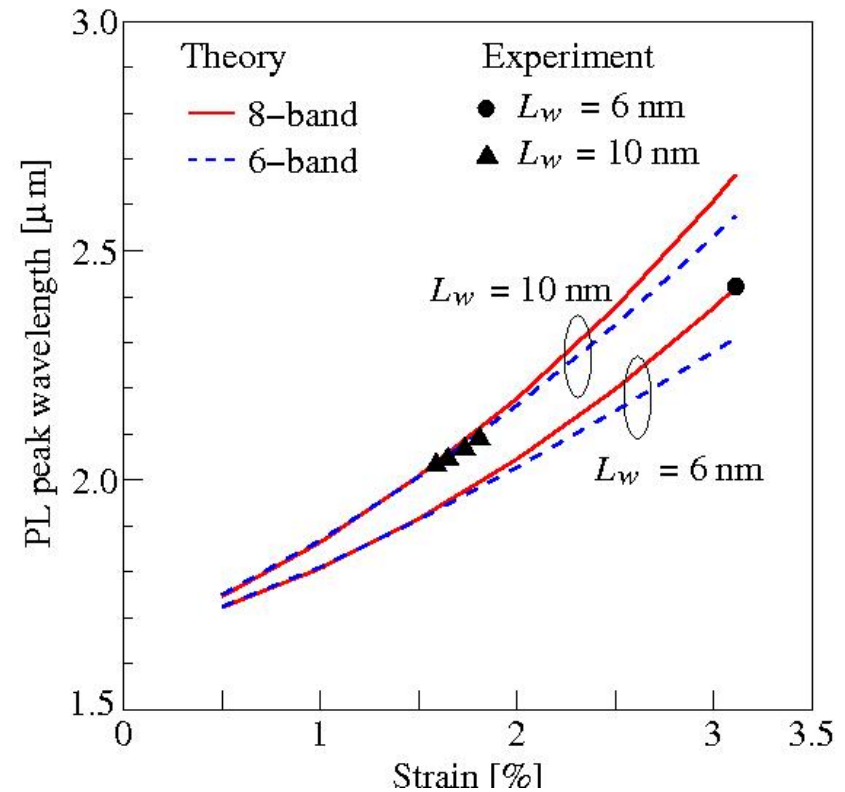
- InAs/InGaAs QWs ($L_w = 6$ nm, $\varepsilon = 3.2\%$)
- 40-nm red shift in calculated PL spectrum
- Excellent agreement with experiment for 8-band model

5-3. Strain dependence of band edge optical properties

Potential profiles of In(Ga)As/InGaAs QWs

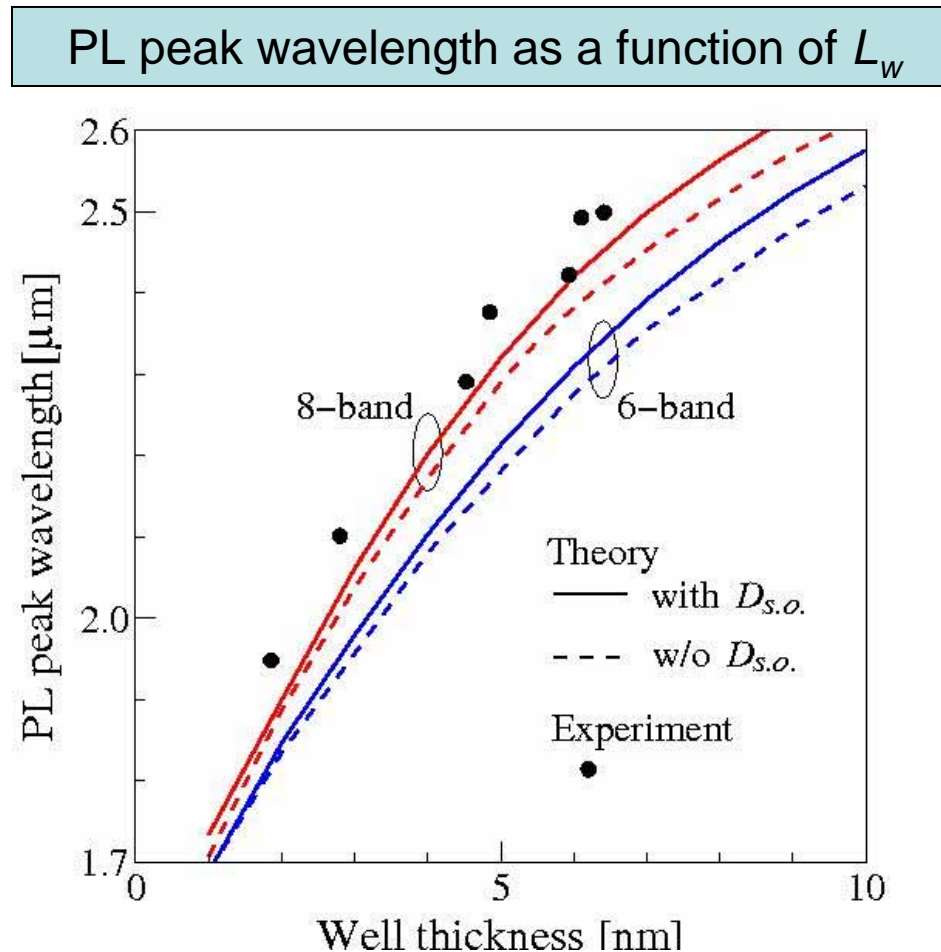


PL peak wavelength as a function of strain



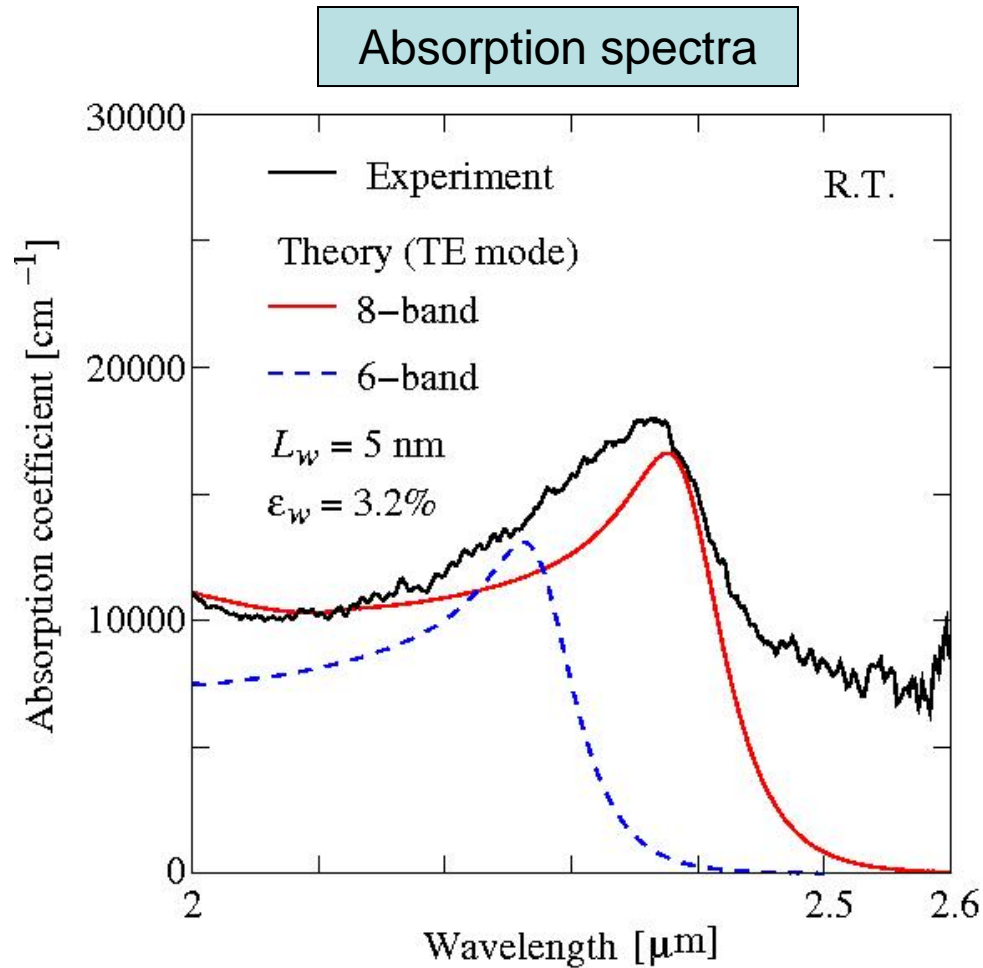
- Stronger quantum confinement for larger strain
- Difference between models becomes large around a strain of 2%.

5-4. Thickness dependence of PL peak wavelength



- InAs/InGaAs QWs ($\epsilon = 3.2\%$)
- > 100 nm too short for 6-band model
- Good agreement between experiment and 8-band model with $D_{s.o.}$

5-5. Absorption spectra



- InAs/InGaAs QWs ($\epsilon = 3.2\%$)
- Good agreement between experiment and 8-band model with $D_{s.o.}$

- 6- and 8-band $\mathbf{k} \cdot \mathbf{p}$ theory
 - Highly strained In(Ga)As/InGaAs QWs (strain up to 3.2%)
- Interaction between strain and spin-orbit coupling ($D_{s.o.}$)
 - 8-band model with $D_{s.o.}$ for InGaAs/InGaAs QWs with the strain larger than 2%

