Simulation of EM wave propagating in a nanocylinder-base localized surface plasma resonance senor

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Outlines

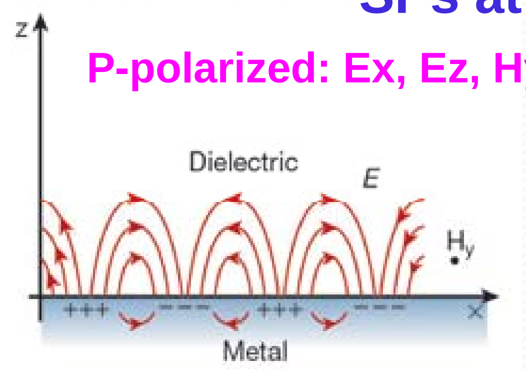
- Introduction of surface plasmon resonance
- Experimental setup
- electronic resonance in single or pair of cylinders
- Field enhancement in Cylinder array
- Sensitivity enhancement
- Magnetic resonance
- conclusions

Introduction

Dispersion Relations of SPs

SPs at metal-dielectric interface

P-polarized: Ex, Ez, Hy



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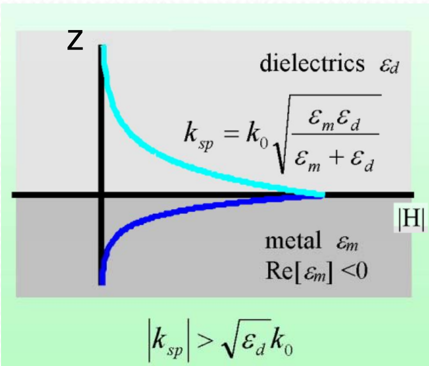
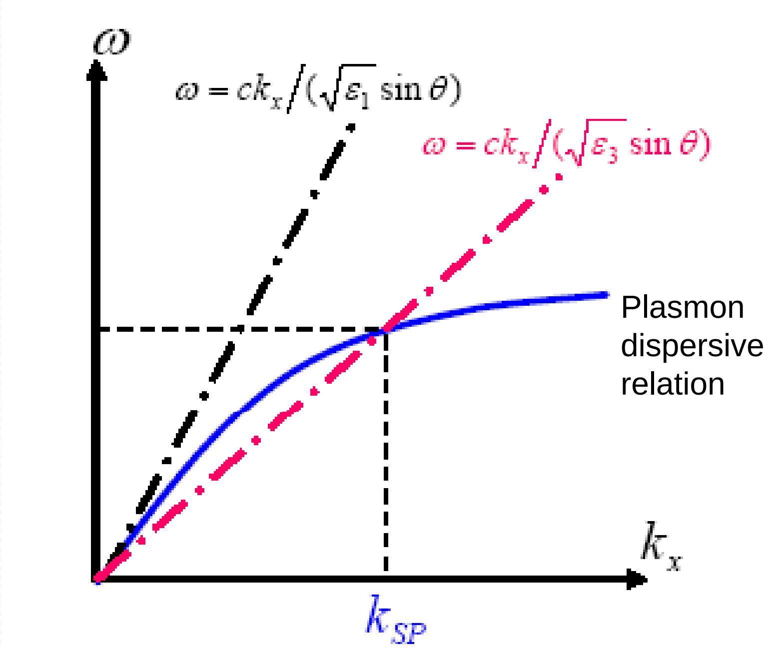
$+\ for\ z \geq 0, -\ for\ z \leq 0$

$k_z : imaginary$



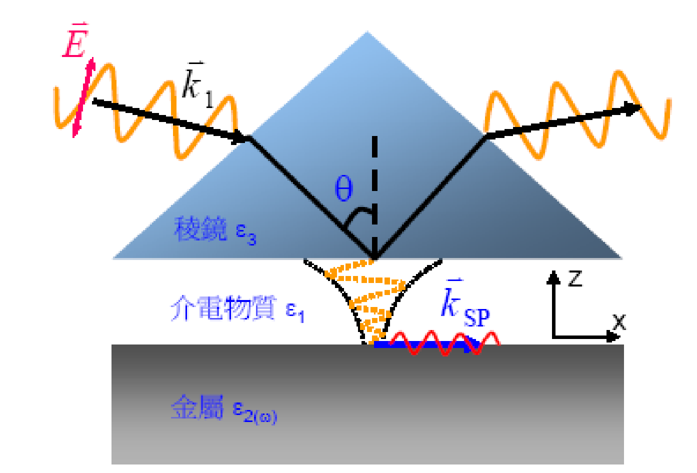
From Maxwell's equations &
E and H fields continuity relations

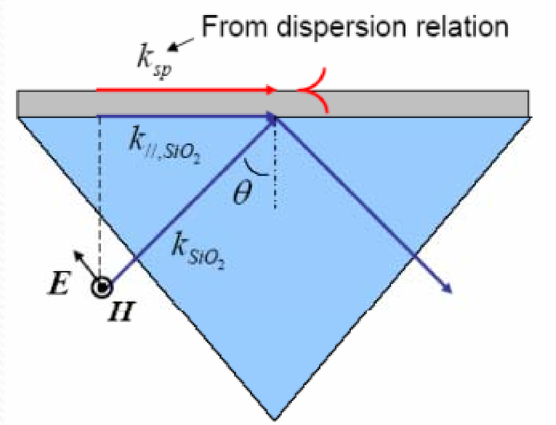
→ **Dispersion relation (DR)**

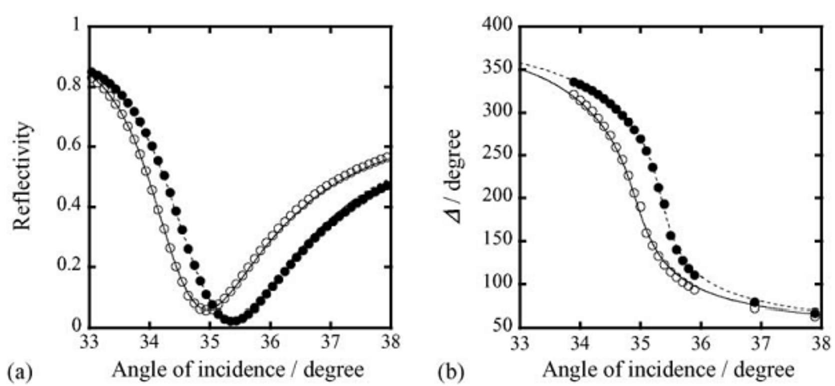


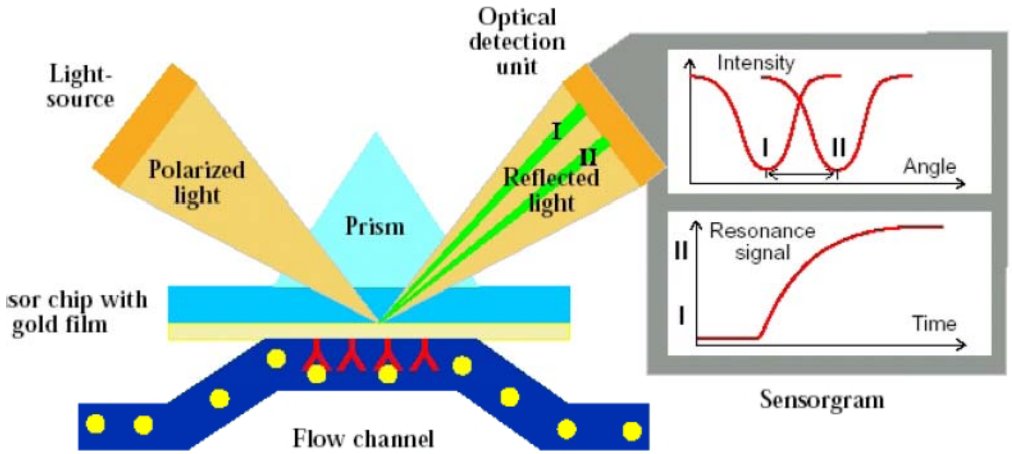
Excitation of SPs

ATR (attenuated total reflection) coupler



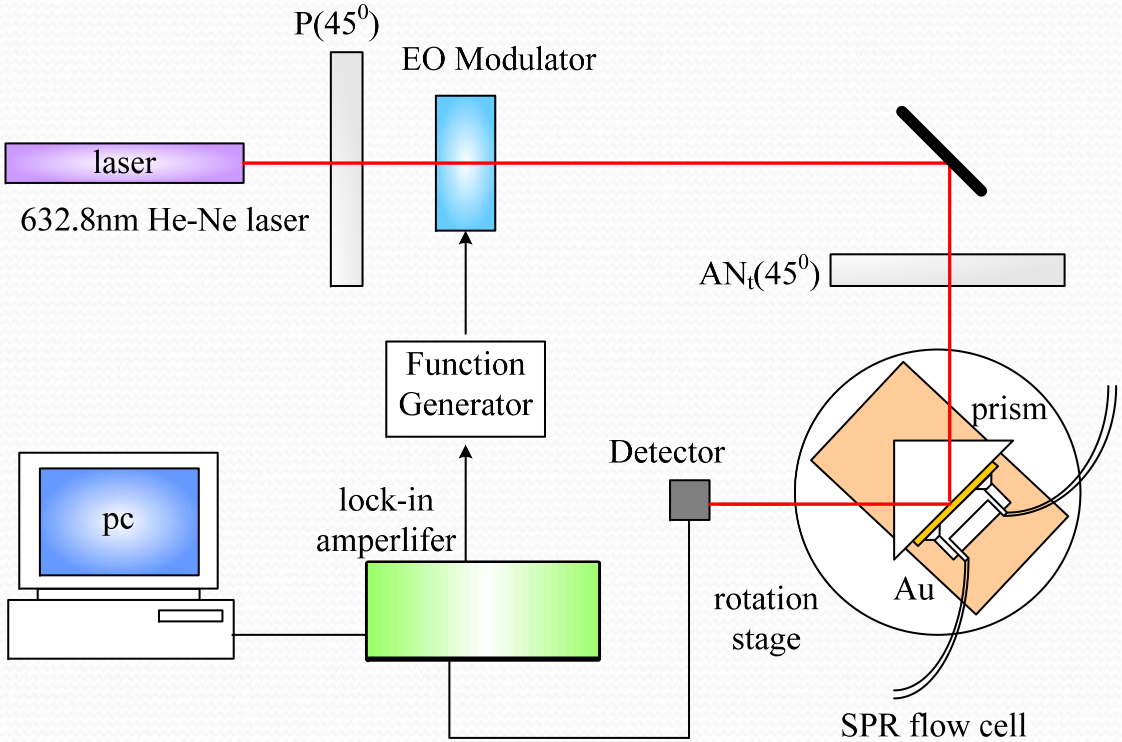






SP sensor

Experimental System



Jones vectors

$$P=\frac{1}{2}\begin{bmatrix}1\\1\end{bmatrix}e^{i\omega_0 t}$$

$$\text{analyzer } AN_t=\frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

$$SPR=\begin{bmatrix}|r_p|e^{i\phi_p} & 0\\ 0 & |r_s|e^{i\phi_s}\end{bmatrix}$$

$$EO=\begin{bmatrix}e^{i\frac{\omega t}{2}} & 0\\ 0 & e^{-i\frac{\omega t}{2}}\end{bmatrix}$$

$$E_t=AN_t\cdot SPR\cdot EO\cdot P \qquad I_t=E_t * E_t$$

$$I_t=\frac{1}{4}\left[\frac{|r_p|^2+|r_s|^2}{2}+|r_p||r_s|\cos\left(\omega t+\phi_p-\phi_s\right)\right]$$

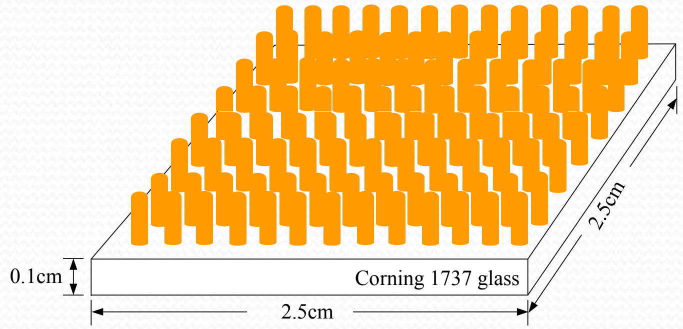
$$\rho=\frac{r_p}{r_s}=\frac{E_{rp}/E_{ip}}{E_{rs}/E_{is}}=\tan\psi\exp\left(i\Delta\right)$$

In general

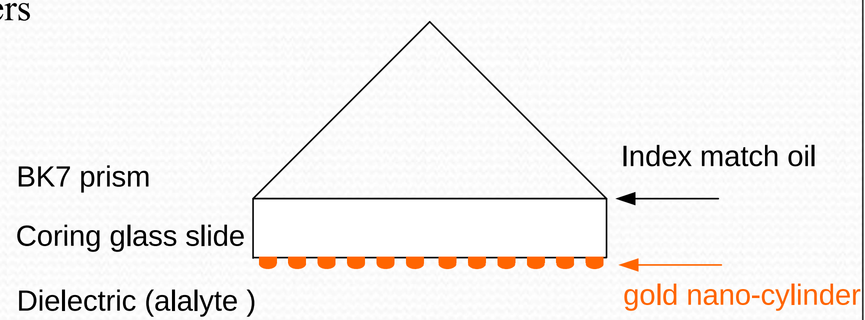
⇨ $|r_s|=1 \quad \phi_s \sim \text{constant}$

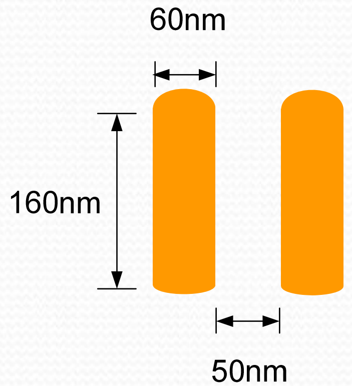
Mode

- Nanoscale cylinder array
  - Interaction between localized and delocalized surface plasmaon polarition modes in a metal photonic crystal (A. Christ etal, *Phys. Stat. sol.* (b) 243. No.10 2344-2348)
  - Plasmon resonance coupling in metallic nanocylinders (Kottmann etal, Optics Express Vol.8 No.12 P655 2001, Sburlan etal, *Physical Review* B 73 035403 2006)
  - Nanowire-based enhancemant of localized surface plasmon resonance for highly sensitive detection (K.Kim, *Optics Express* Vol.14, No.25, 12419 2006)
  - Resonance light interaction with plasmonic nanocylinder system (ref: Viktor A. Podolskiy etal *J. of Optics A* 7 P32-37 2005)



Nano-cylinder array





Macroscopic permittivities

- Maxwell's equation in the metallic cylinder

$$\nabla \cdot \mathrm{D} = 4\pi\rho = \frac{4\pi}{\mathrm{i}\omega}\nabla \cdot \mathrm{J} = \nabla \cdot \left(\frac{4\pi\sigma}{\mathrm{i}\omega}\mathrm{E} \right) \Rightarrow \nabla \cdot (\varepsilon \mathrm{E}) = 0 \qquad \varepsilon \equiv \varepsilon_\infty - \frac{4\pi\sigma}{i\omega}$$

$$\langle M \rangle = \frac{1}{2cV}\int d^3 r \theta_1(r)\left[r \times \left(J(r) - \frac{i\omega}{4\pi} D(r) \right) \right] = -\frac{i\omega}{8\pi c}\varepsilon_1 \int d^3 r \theta_1(r)\left[r \times E(r) \right]$$

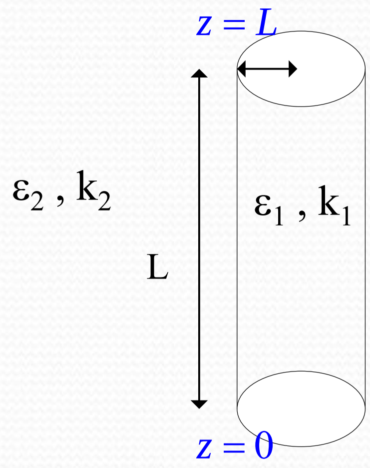
$$\langle B \rangle = \frac{c}{\omega}\left(k \times \langle E \rangle \right) = \sqrt{\varepsilon_e \mu_e}\left(e_k \times \langle E \rangle \right) \equiv \mu_e \langle H \rangle = \mu_e \langle B - 4\pi M \rangle \qquad \langle \varepsilon E \rangle \equiv \varepsilon_\mathrm{e} \langle E \rangle$$

$$\Rightarrow \langle M \rangle = \frac{\mu_e - 1}{4\pi} B = \frac{\mu_e - 1}{4\pi}\sqrt{\varepsilon_e \mu_e}\left(e_k \times \langle E \rangle \right) \qquad \varepsilon_\mathrm{e} \equiv \varepsilon_\infty \left(1 - \frac{\omega_\mathrm{p}^2}{\omega^2 - \mathrm{i}\omega\gamma} \right)$$

- Enhanced E-field make contribution ε_e, but μ_e limited.

Mathematatic Description

- Single cylinder
  - $L>>a$, $\lambda>L$, the standing waves are good approximation to the eigenstates: $|M|\geq 1$



$$E_M(r) = -\cos(k_z z)\sum_{q=\pm 1}(q + signM \;\; k_z^2/k_2^2)e_q J_{M\text{-}q}(\alpha\rho)e^{i(M\text{-}q)\varphi}$$

$$+\sin k_z z\sqrt{2}\alpha\left(k_z/k_2^2\right)signM \;\; e_0 J_M(\alpha\rho)e^{iM\varphi}$$

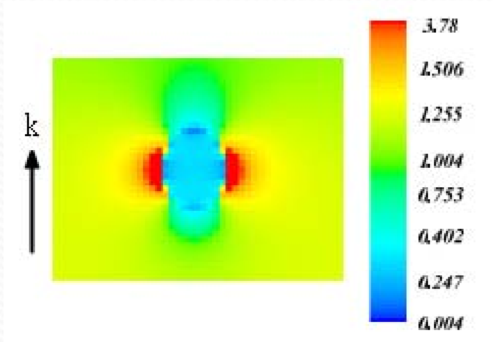
B.C.

$E_M = 0$ at $z = 0, L$

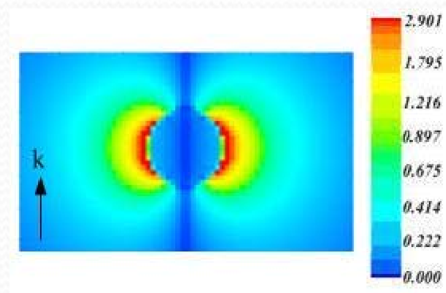
$$e_{\pm 1} \equiv \mp\frac{e_x \pm ie_y}{\sqrt{2}},\; e_0 = e_z;\; k_i^2 \equiv \varepsilon_i\frac{\omega^2}{c^2} = \alpha_i^2 + k_z^2,\; i = 1, 2$$

$$k_z L = n\pi,$$

$$n = k_2 z/\pi,\; k_2 z/\pi + 1,\; \ldots$$

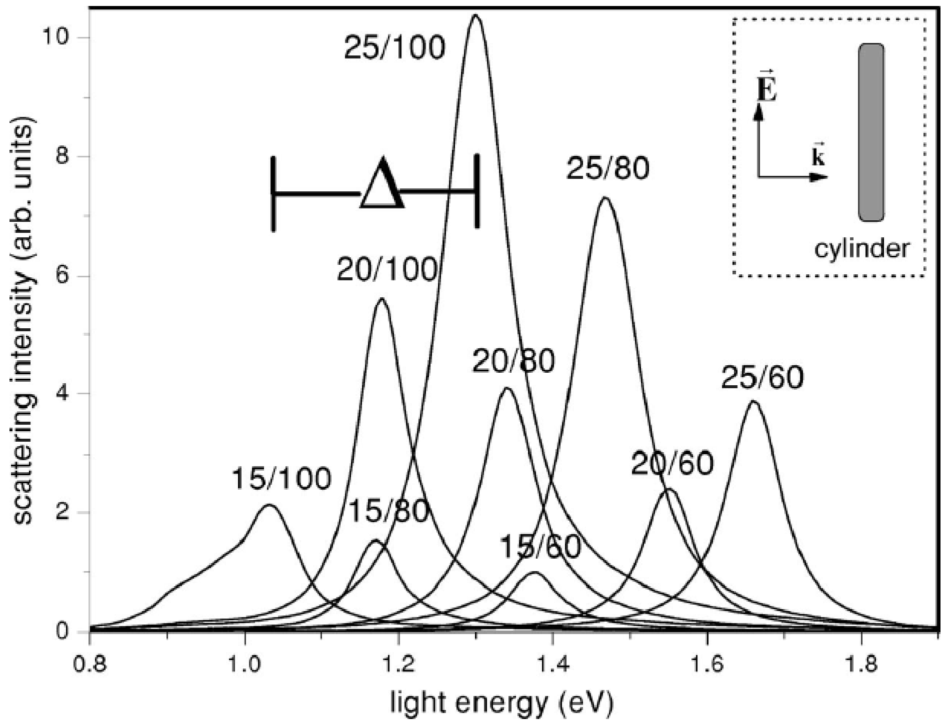


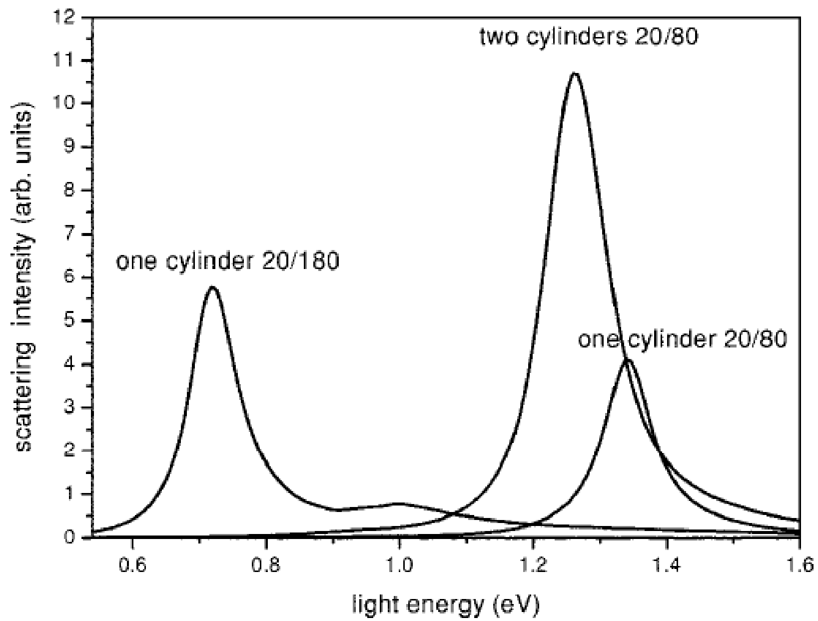
TM Mode



TE Mode

- TM mode (p-wave) field enhancement is a function of cylinder dimension



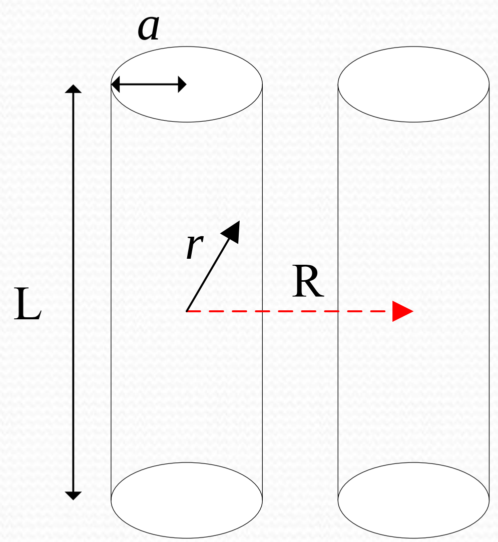


S.E. Sburlan Physical Review B 73. 035403 2006

- Two cylinders
  - $\rho < a$

$$E_M(r) = -\cos(k_z z)\sum_{q=\pm 1}(q + signM\ \ k_z^2/k_2^2)e_q J_{M-q}(\alpha_1 \rho)e^{i(M-q)\varphi}$$

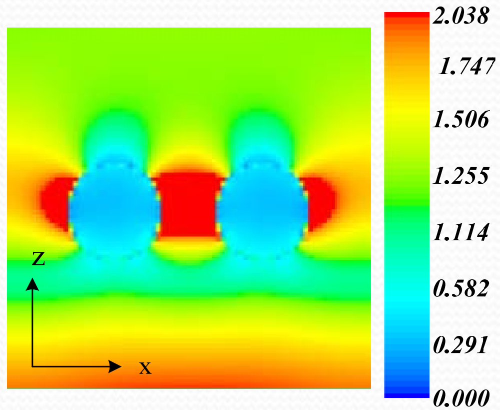
$$+\sin k_z z\sqrt{2}\alpha_1(k_z/k_2^2)\,signM\ \ e_z J_M(\alpha_1\rho)e^{iM\varphi}$$



  - $\rho > a$

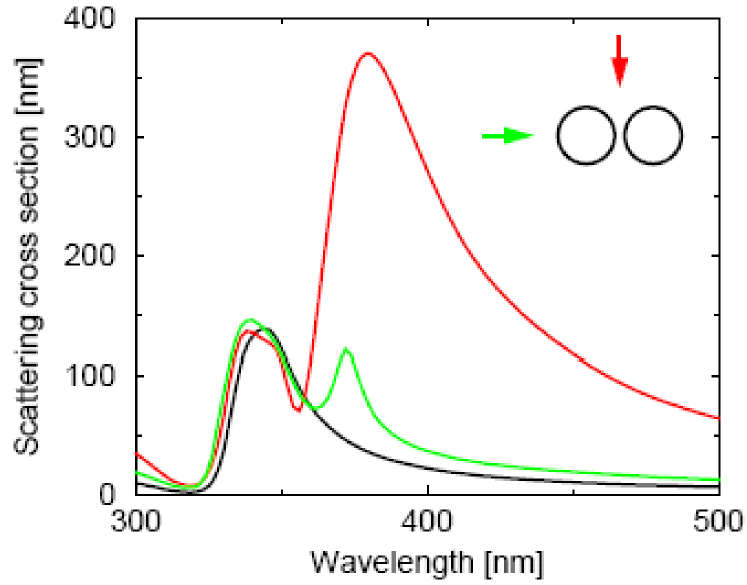
$$E_M(r) = -\cos(k_z z)\sum_{q=\pm 1}(q + signM\ \ k_z^2/k_2^2)e_q Y_{M-q}(\alpha_2 \rho)e^{i(M-q)\varphi}$$

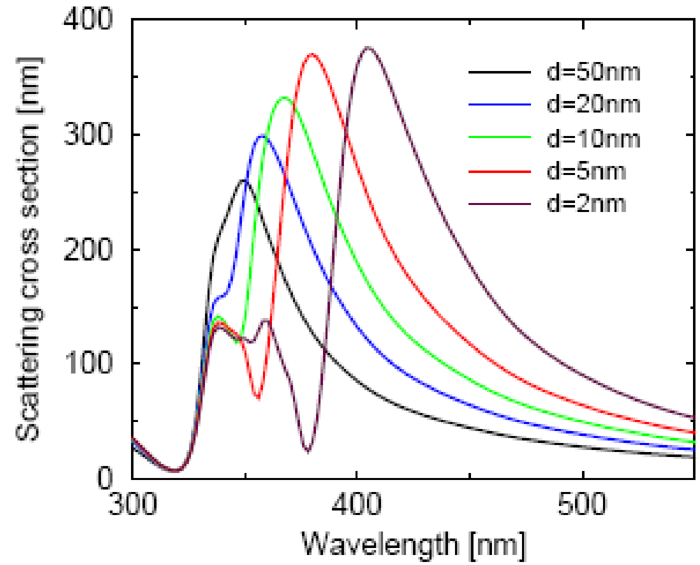
$$+\sin k_z z\sqrt{2}\alpha_2(k_z/k_2^2)\,signM\ \ e_z Y_M(\alpha_2\rho)e^{iM\varphi}$$



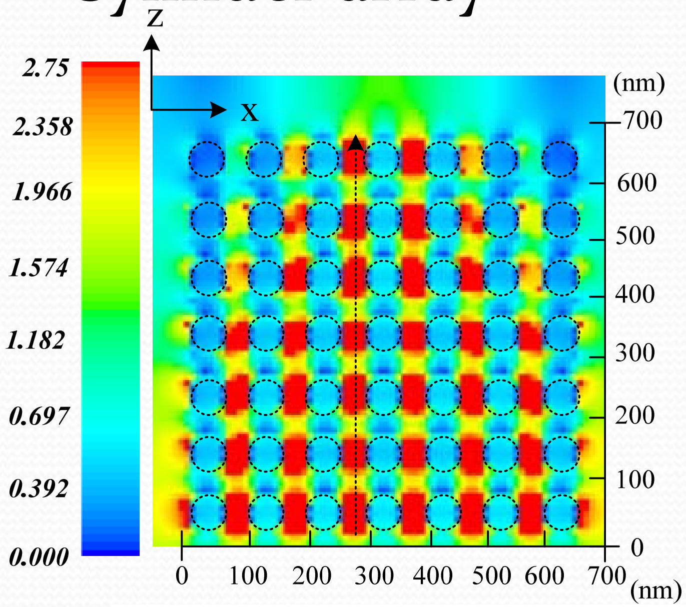
- Coupled system has the same magnitude resonance as for the individual cylinder, but the coupling is much stronger and depends on the incident field

- Main resonance is red-shifted with decreasing distance d

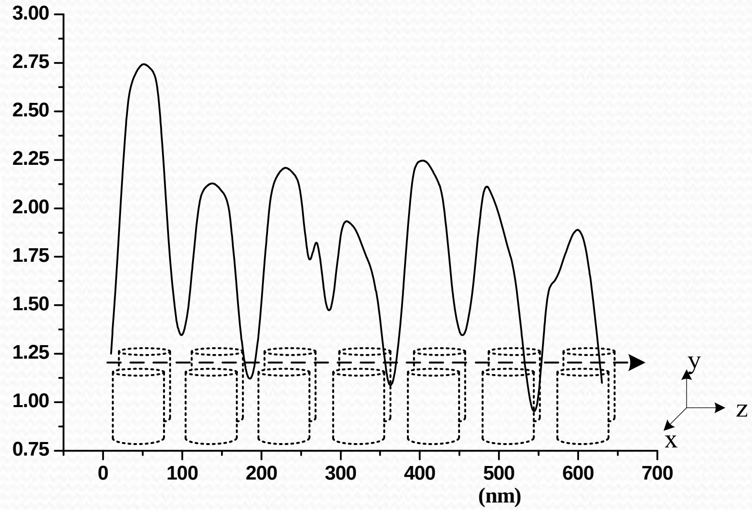




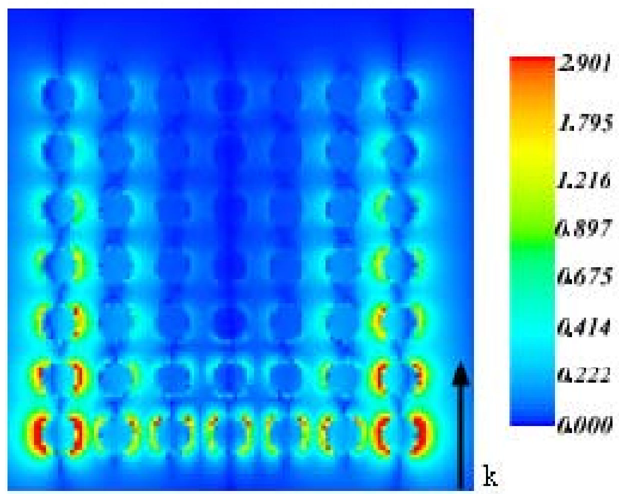
Cylinder array

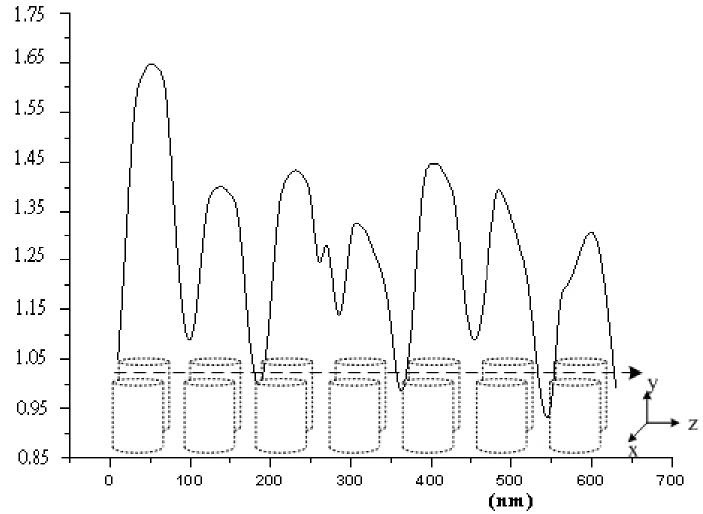


TM Mode



TE Mode





Local Surface Plasmon Enhance Sensitivity

- Sensitivity Definition: Mathematic formula

$$s = \frac{\Delta h}{\Delta n}$$

Δn : Refraction index variation

Δh : Physical parameter variation (θ or λ)

- Resolution

$$\sigma_{\mathrm{RI}} = \frac{\sigma_{ins}}{s}$$

σ_{ins} : Instrument Resolution

- Present

σ_{RI} :

Can reach ~ 10^{-7} for using heterodyne interferometer

Interrogation	Angular	Wave	Intensit y	Phase
σ_{ins}	0.0001°	0.02nm	0.2%	0.01°
Prism coupling	5×10^{-7}	2×10^{-5}	5×10^{-5}	4.6×10^{-7}
Grating coupling	2×10^{-6}	6×10^{-5}	2×10^{-4}	

J. Homola, S.S.Yee and G.Gauglitz "Surface plasma resonance sensors: Review" Sens. Actuators B 54, 3-15 (1999)

- Local surface plasma resonance is accompanied by the broadening in dispersion relation which increase the damping term γ *
- Surface plasmon resonance consists of gold thin film part and nanocylinder part, angular width is

$$\Delta\theta_{sp} = \frac{\mathrm{Im}\{k_{sp}\}}{n_s \frac{\omega}{c}\cos\theta_{sp}} = \frac{\gamma_i + \gamma_r}{n_s \frac{\omega}{c}\cos\theta_{sp}}$$

$$\varepsilon_e \equiv \varepsilon_\infty \left(1 - \frac{\omega_p^2}{\omega^2 - i\omega\gamma}\right) = \varepsilon'_{ca} + i\varepsilon''_{ca}$$

$$\mathrm{Im}\{k_{sp}\} \approx \frac{\omega}{c}\left(\frac{\varepsilon'_{Au}\varepsilon'_{ca}}{\varepsilon'_{Au} + \varepsilon'_{ca}}\right)\frac{\varepsilon''_{Au}}{2\left(\varepsilon'_{Au}\right)^2}$$

$$\left|\varepsilon'_{Au}\right| \gg \varepsilon''_{Au} \qquad \varepsilon''_{ca} \approx 0$$

Metallic photonic crystal

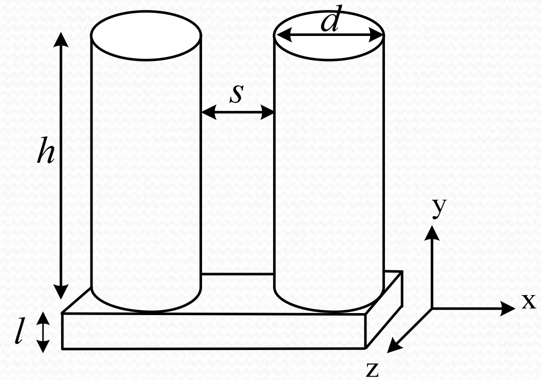
$$\Delta\theta_{sp} = \frac{2}{n_s\cos\theta_{sp}}\left(\frac{\varepsilon'_{Au}\varepsilon'_{ca}}{\varepsilon'_{Au} + \varepsilon'_{ca}}\right)\frac{\varepsilon''_{Au}}{2\left(\varepsilon'_{Au}\right)^2}$$

$$\left|\varepsilon'_{Au}\right| \sim \varepsilon'_{ca}$$

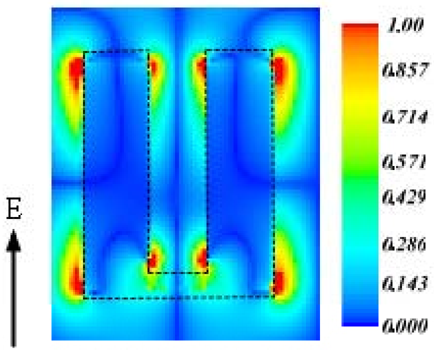
* Kyujung Kim Optics Express Vol 14, No 25 12419 (2006)

Magnetic Resonance

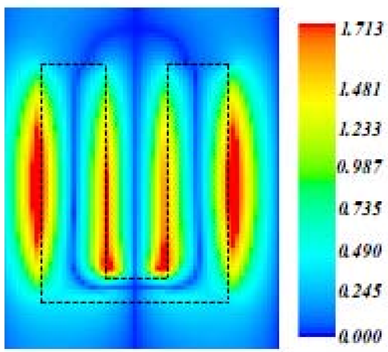
- TE Mode:
  - Anti-symmetric polarition mode
  - Anti-parallel current in the two cylinders induce magnetic dipole moment resonance



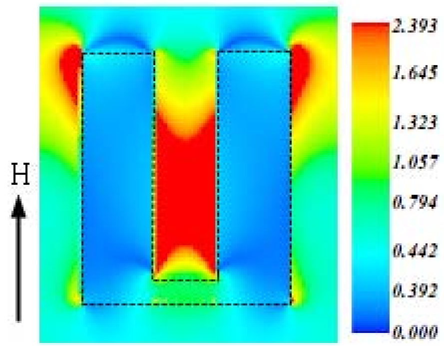
- TM Mode: Symmetric polarition will induce the electric resonance, but magnetic resonance is limited



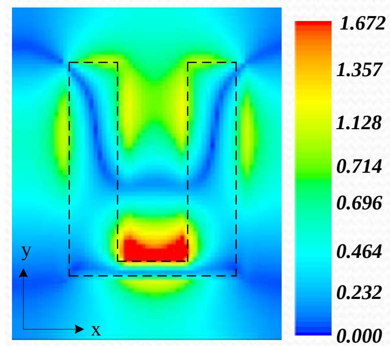
E in TE mode



H in TE mode



E in TM mode



H in TM mode

Conclusions

- Significant sensitivity increase is associated with larger field enhance.
- Field enhancement is achieved by nanocylinder based localized plasmon resonance mediated by nanocylinder and their coupling.
- Coupling has more contribution on the field enhance.
- Significant magnetic field enhancement is happen in TE mode due to anti-symmetric polarition mode.